Orbits around black holes in triaxial nuclei

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We discuss the properties of orbits within the influence sphere of a supermassive black hole (BH), in the case that the surrounding star cluster is nonaxisymmetric. There are four major orbit families; one of these, the pyramid orbits, have the interesting property that they can approach arbitrarily closely to the BH. We derive the orbit-averaged equations of motion and show that in the limit of weak triaxiality, the pyramid orbits are integrable: the motion consists of a two-dimensional libration of the major axis of the orbit about the short axis of the triaxial figure, with eccentricity varying as a function of the two orientation angles, and reaching unity at the corners. Because pyramid orbits occupy the lowest angular momentum regions of phase space, they compete with collisional loss cone repopulation and with resonant relaxation in supplying matter to BHs. We derive expressions for the capture rate, including the effects of general relativistic precession which imposes an upper limit to the eccentricity. We show that capture from pyramid orbits can dominate the feeding of BHs, particularly in giant galaxies, at least until such a time as the pyramid orbits are depleted; however this time can be of order a Hubble time.

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Introduction

We consider the motion of stars in the dominant potential of point mass $M_\bullet$ with addition of potential from a distributed mass of stellar cusp, which is (weakly) triaxial. The orbit of a star in the vicinity of a supermassive black hole (BH) may be regarded as perturbed Keplerian motion. On short timescale ($\sim P$ – the radial period) orbit is an almost closed ellipse, whose orientation changes more slowly. We employ orbit-averaging technique (1; 2) to reduce the problem to two-dimensional Hamiltonian system with the action variables are ℓ and ℓ_z – angular momentum and its z component (normalized to the angular momentum of a circular orbit). This system is integrable in the case of small triaxiality ϵ. There are four classes of orbits (Fig.1), three of them being tubes and one is a Keplerian analogue of box orbits – the pyramid orbits (discovered in (3; 4), and thoroughly studied in the present paper).

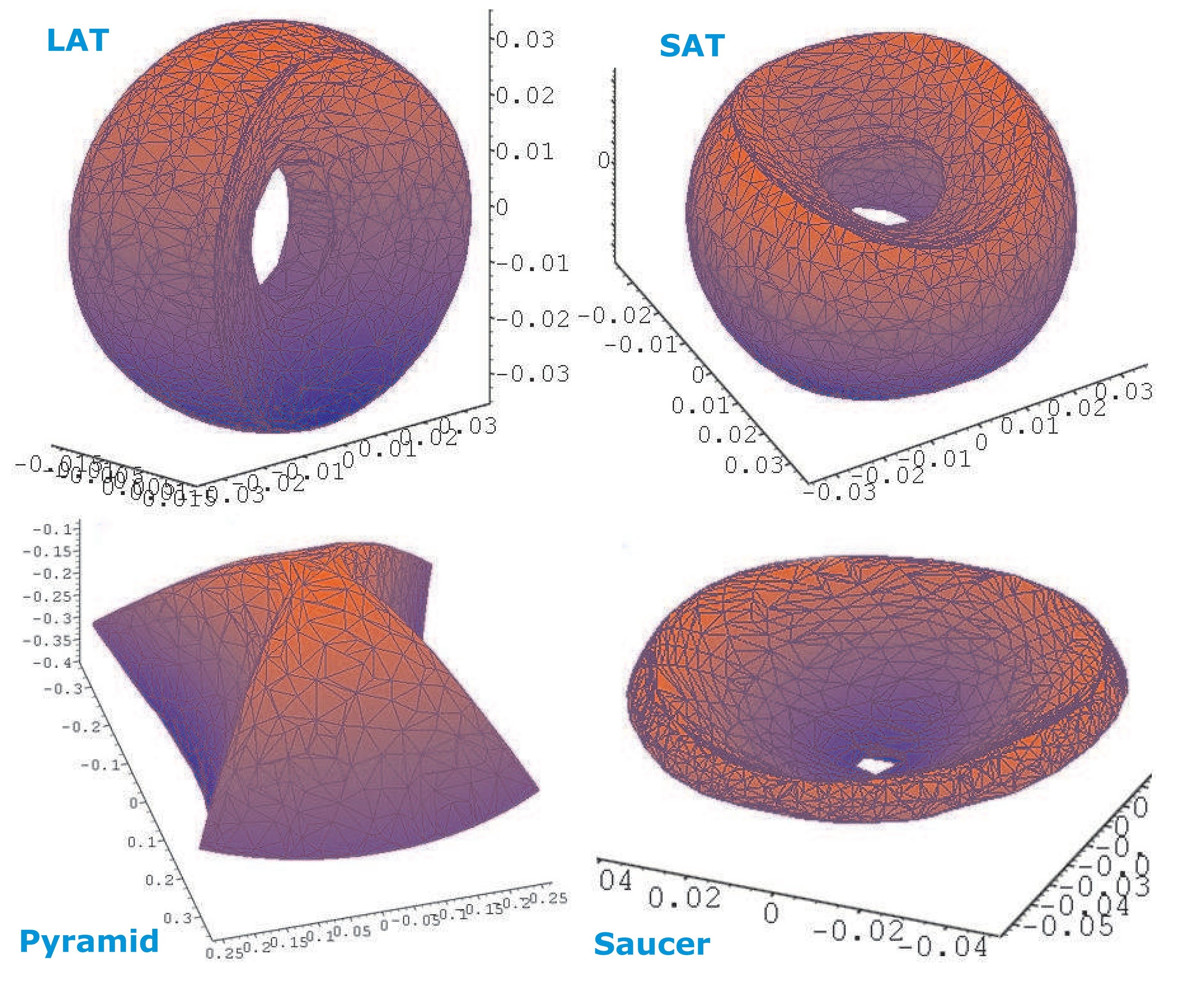

Fig.1: Four types of orbits in triaxial nuclei: long- and short-axis tubes, saucers (subfamily of SAT) and pyramids.

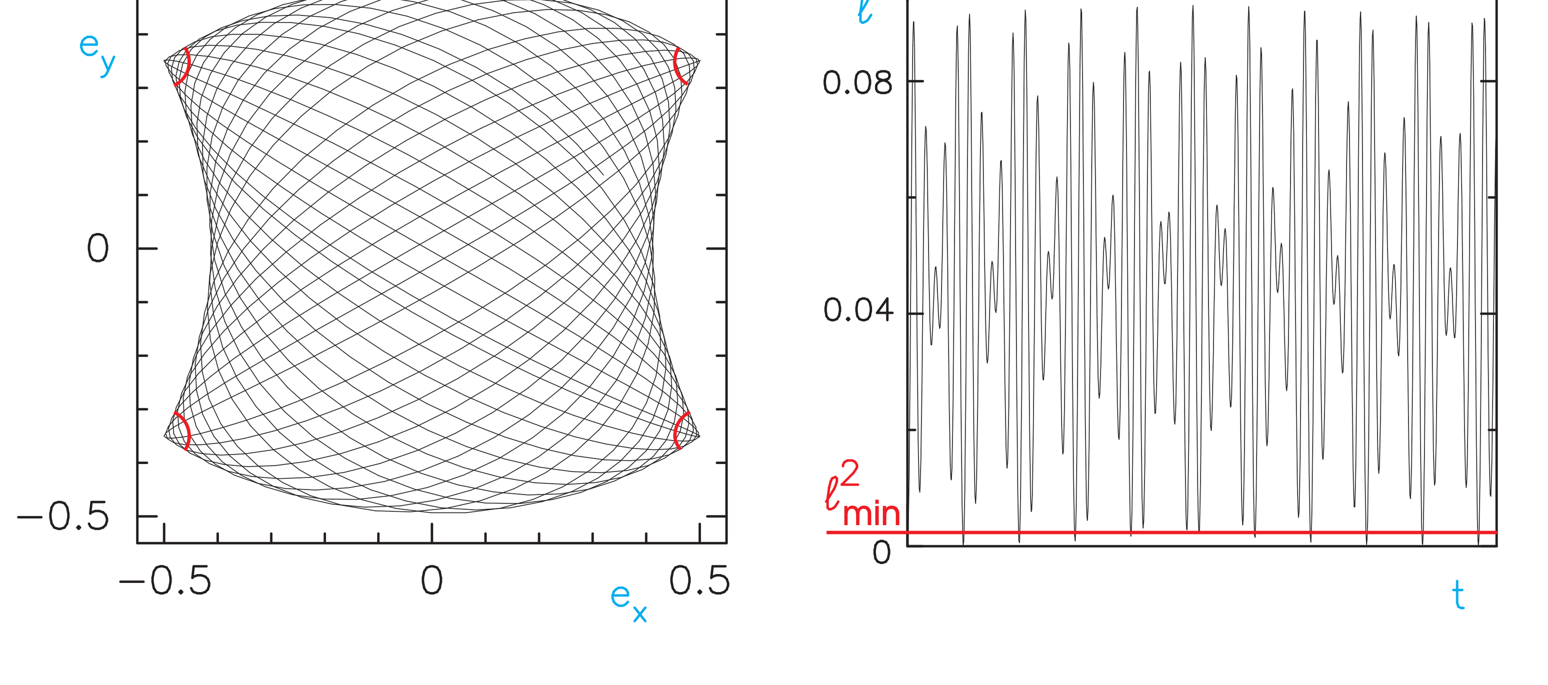

Fig.2: Left: "base" of a pyramid orbit (components of eccentricity vector e_x, e_y); Right: time evolution of squared angular momentum ℓ^2. Red line marks the critical value of ℓ below which a star may be disrupted by the BH; it corresponds to region near corner points of pyramid orbit.

Pyramid orbits

The non-axisymmetric potential leads to non-conservation of both angular momentum ℓ and its z component ℓ_z. They oscillate with characteristic frequency $\nu \sim 2\pi P^{-1}\sqrt{\epsilon}\,(M_\star(r)/M_\bullet)$, where $M_\star$ is the distributed (stellar) mass within the radius of the orbit. The orbit-averaged method is applicable only for radii $r \lesssim r_{bh}$, where the BH influence radius r_{bh} is defined to contain $M_\star \simeq M_\bullet$. The pyramid orbits are best described in terms of the eccentricity vector $\mathbf{e}$. It corresponds to the vector in physical space, directed from the BH to the apoapse of the orbit. Its components e_x and e_y perform oscillations (Fig.2, left), which are harmonic and uncoupled in the small-amplitude limit $e_{x,y} \ll 1$. The vector e_x, e_y fills a rectangular region, which defines the "base" of the pyramid orbit in physical space. The squared angular momentum ℓ^2 oscillates from a maximum value $\ell^2_{max} \sim \epsilon$ down to 0 (Fig.2, right).

Since the BH captures or tidally disrupts stars with angular momentum less than a certain minimal value ℓ_{min}, the pyramid orbits are a good candidate for feeding the BH. Approximately $(\ell_{min}/\ell_{max})^2$ stars are lost during each radial orbital period P, and the time for fully depleting all pyramid orbits is typically less than the Hubble time (Fig.3).

The effects of general relativity introduce additional periapse precession, which leads to nonzero lower limit on ℓ which a pyramid orbit may attain. For $r \gtrsim 0.3 r_{bh}$ this lower limit is still less than critical value for capture, and pyramids may be consumed by the BH. At $r \gtrsim r_{bh}$ most low-ℓ orbits are turned to chaotic (which still may contribute to feeding rates, see (4)).

The total capture rate of stars on pyramid orbits is $\sim 10^{-3}\,M_\odot \mathrm{yr}^{-1}\,(M_\bullet/10^8\,M_\odot)$, which is much larger than the capture rate due to two-body relaxation. The lifetime of pyramid orbits at $r \sim r_{bh}$ is $\sim 10^{11}\epsilon$ yr.

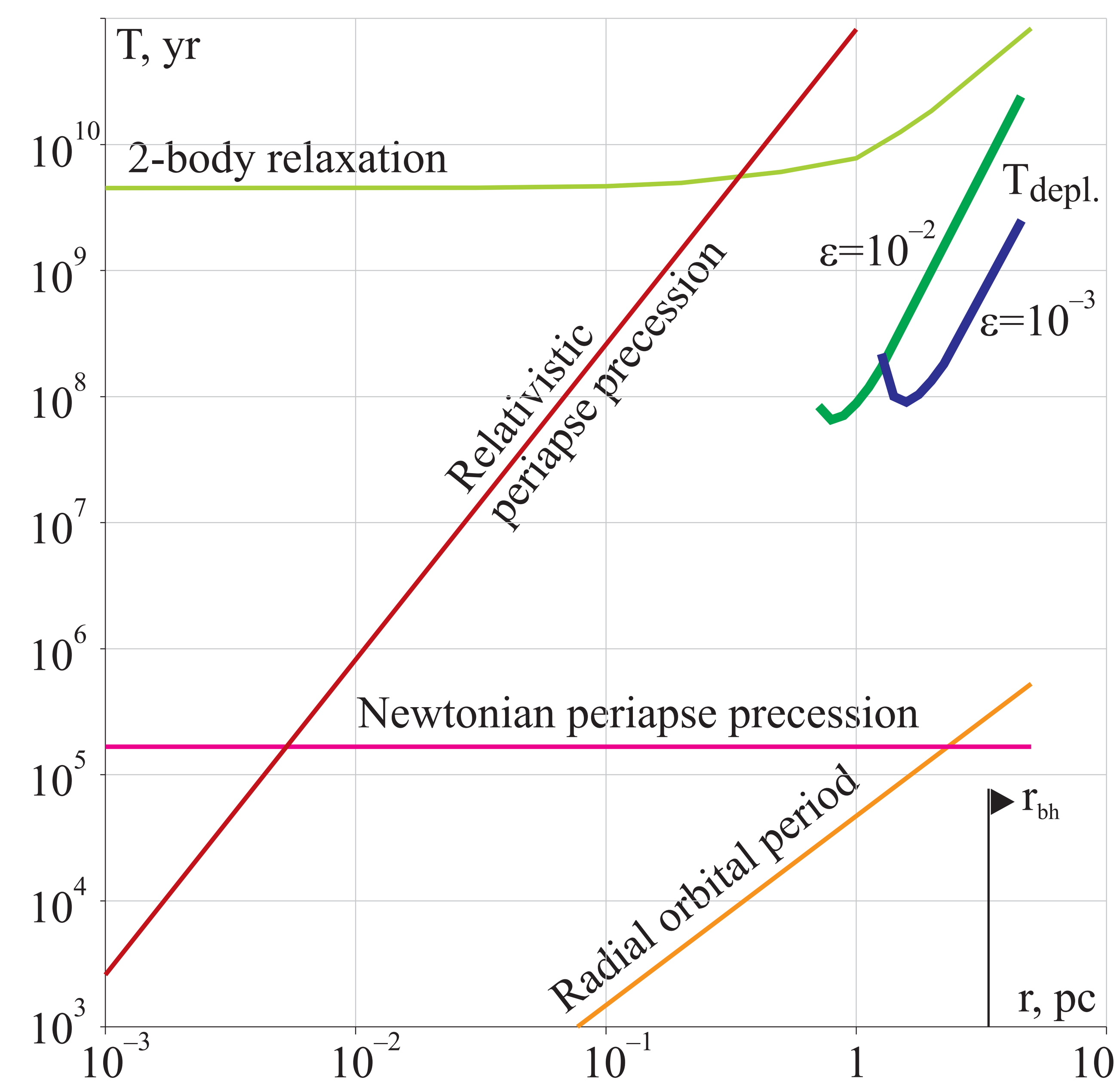

Fig.3: Timescales of various processes in the centre of Milky Way. $r_{bh} \simeq 3$ pc is the black hole influence radius; T_{depl} is the timescale for pyramid orbit depletion; cutoff at low radius is due to the influence of relativistic precession.

References

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