DYNAMICAL MODELLING OF BARRED GALAXIES

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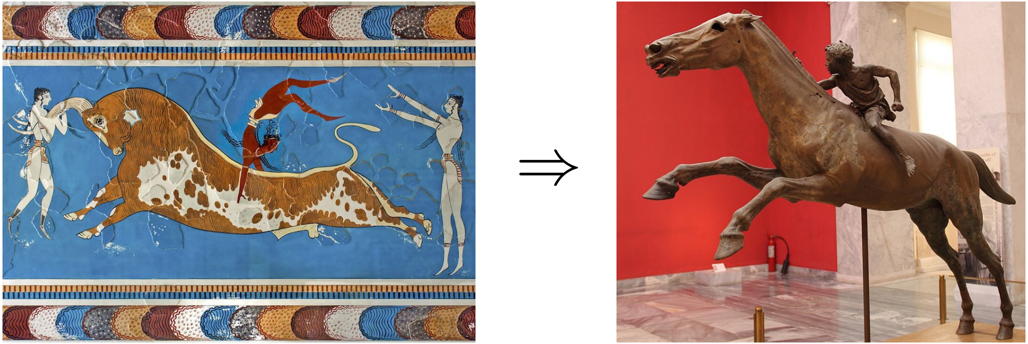
Academy of Athens, 23 April 2024

Synopsis

- Motivation
    - Barred galaxies are ubiquitous ($\sim$ 50% of disk galaxies)
    - Complex morphology and kinematics
    - Interplay between bars and supermassive black holes
- The problem of deprojection
    - Non-uniqueness
    - Forward modelling of 3d shapes
- Modelling approaches for barred galaxies
    - Overview of methods
    - Schwarzschild's method in brief
- First results for barred galaxies and supermassive black holes

The problem



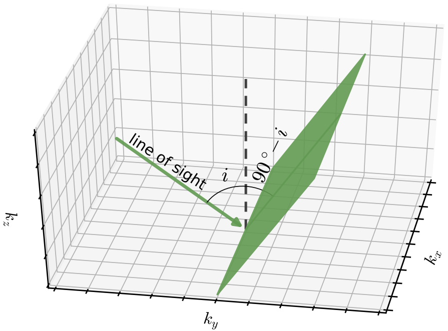
Pathway from 2d surface brightness profile to 3d density profile is non-unique

The problem

Fourier Slice Theorem [Rybizki 1987]:

surface density $\Sigma(X, Y) \implies$ its Fourier transform $\hat{\Sigma}(k_X, k_Y)$ corresponds to the Fourier transform of the 3d density $\hat{\rho}(k_X, k_Y, k_Z = 0)$, i.e. provides **no** constraints on $\hat{\rho}(\ldots, k_Z \neq 0)$.

For an axisymmetric system at an inclination i, nothing is known of its $\hat{\rho}$ in the “cone of ignorance” with opening angle $90^\circ - i$ around k_z.



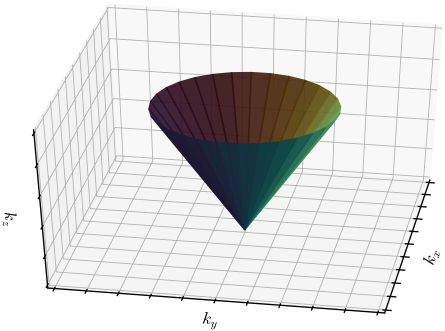
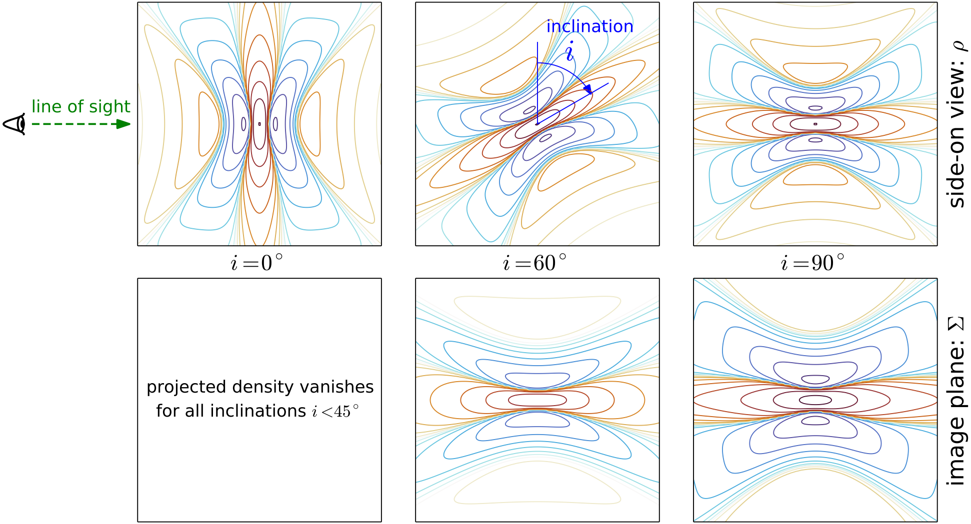


Illustration of non-uniqueness of axisymmetric deprojection

It turns out that there is a large family of axisymmetric “konus density” profiles that are completely invisible at any inclination $i \leq i_{\text{min}} < 90^\circ$ [Gerhard & Binney 1996; Kochanek & Rybizki 1996].



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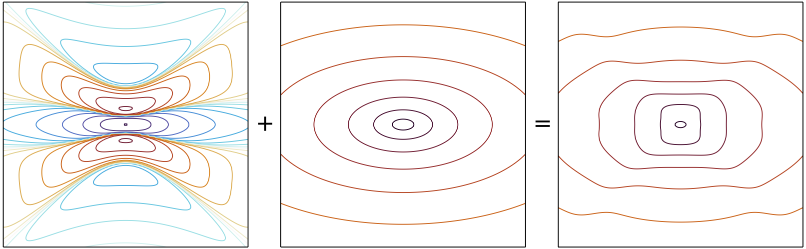
Adding it to an ordinary ellipsoidal density profile, one can make it boxy or disky, while still appearing perfectly elliptical in projection.

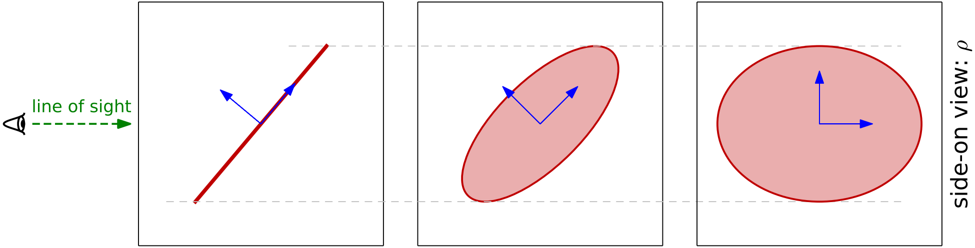


The degeneracy is **much** worse for triaxial systems.

Adding kinematic information should lift the degeneracy [Magorrian 1999].

Approaches to deprojection

1. Ellipsoidal assumption: $\rho(x, y, z) = \rho(m),\ m \equiv \sqrt{x^2 + (y/p)^2 + (z/q)^2}$.



$i = 40^\circ,\ \ q = 0.00$ $i = 45^\circ,\ \ q = 0.42$ $i = 90^\circ,\ \ q = q' = 0.77$

In the axisymmetric case, the projected axis ratio $q' = \sqrt{q^2\ \sin^2 i + \cos^2 i}$
$\Longrightarrow$ deprojection is possible for inclination angles $i > i_{\min} \equiv \arccos q'$.

Generalization to a triaxial case: for a given projected shape and assumed orientation (viewing angles), the deprojection is either *unique* or impossible.

Widely used in practice, e.g. Multi-Gaussian Expansion [Cappellari 2002].

Approaches to deprojection

1. Ellipsoidal assumption: $\rho(x, y, z) = \rho(m),\ m \equiv \sqrt{x^2 + (y/p)^2 + (z/q)^2}$.
2. Forward modelling of projected density:

parametric

- choose a suitable functional form for $\rho(x, y, z;\ \mathbf{p})$
- assume some viewing angles $\boldsymbol{\psi}$ and parameters $\mathbf{p}$
- compute the projected profile $\Sigma(X, Y)$
- compare with the observed surface density and compute deviation χ^2
- repeat for different choices of $\boldsymbol{\psi}$ and $\mathbf{p}$ to minimize χ^2

non-parametric

(or rather, multiparametric)

- choose a *very general / flexible* functional form with many free params (e.g., splines or a basis set expansion)
- assume some angles $\boldsymbol{\psi}$ and params $\mathbf{p}$
- compute projected Σ
- compare with observations; compute χ^2 and add some regularization penalty
- repeat for many choices of $\boldsymbol{\psi}$ and $\mathbf{p}$

[e.g., Magorrian 1999; de Nicola+ 2020]

Photometric fitting

[reasonably] simple models

multiple components

sky subtraction

foreground masking

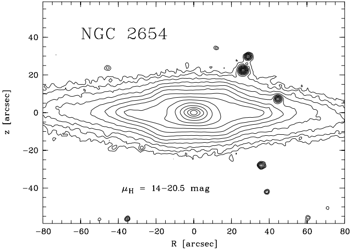
PSF convolution

- MGEFit [Cappellari 2002]:
  "nonparametric" (multiple elliptical Gaussians) ⇒ ellipsoidal deprojection
- GalFit [Peng+ 2002, 2010]: many flexible 2d profiles,
  but deprojection is straightforward only for ellipsoidal models
- ImFit [Erwin 2015]: many 2d **and 3d profiles** (including user-defined),
  may project 3d model to 2d instead of deprojecting 2d to 3d

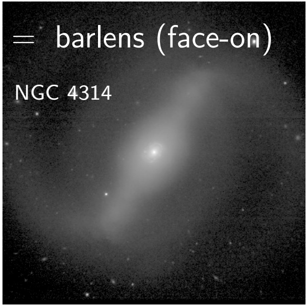
3D structure of bars

Bars often buckle vertically from the disk plane, but only in the inner part where the planar orbits are unstable;
shorter and vertically thick part is associated with boxy/peanut (B/P) bulges, and the longer and thinner component can be seen in face-on barlens galaxies [Athanassoula 2005, 2013].

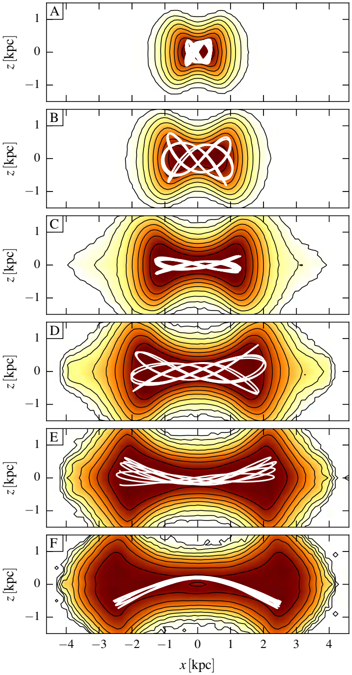
boxy/peanut bar (edge-on) = barlens (face-on)



[Lütticke+ 2000]

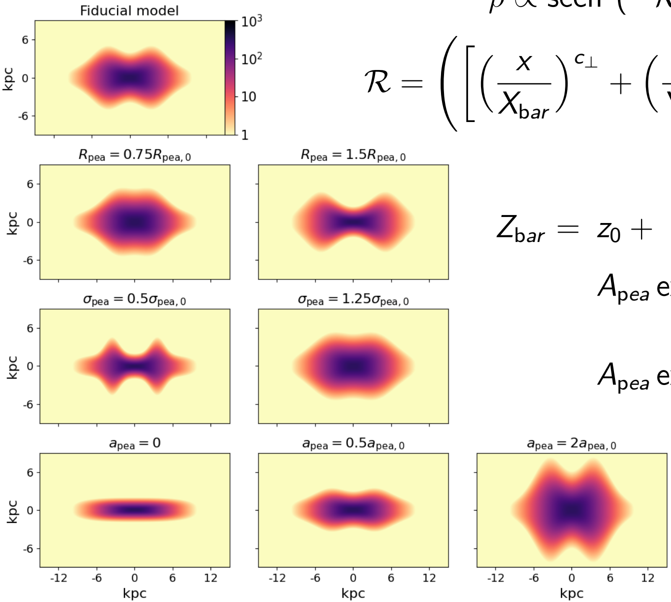


[Laurikainen+ 2011]



[Portail+ 2015]

X-shaped bar model [Robin+ 2012; Fragkoudi+ 2015]



$$\rho \propto \mathrm{sech}^2(-\mathcal{R})$$

$$\mathcal{R} = \left(\left[\left(\frac{x}{X_{\rm bar}} \right)^{c_\perp} + \left(\frac{y}{Y_{\rm bar}} \right)^{c_\perp} \right]^{c_\parallel / c_\perp} + \left(\frac{z}{Z_{\rm bar}} \right)^{c_\parallel} \right)^{1/c_\parallel}$$

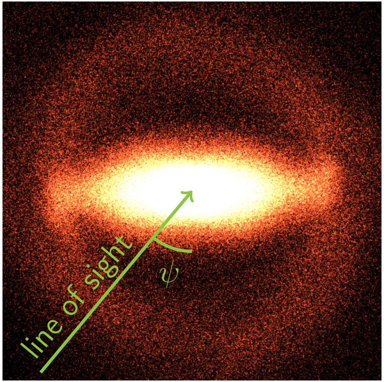
$c_\parallel$, $c_\perp$: boxiness coefficients

$$Z_{\rm bar} = z_0 + A_{\rm pea} \exp\left(- \frac{(x - R_{\rm pea})^2 + y^2}{2 w_{\rm pea}^2} \right) + A_{\rm pea} \exp\left(- \frac{(x + R_{\rm pea})^2 + y^2}{2 w_{\rm pea}^2} \right)$$

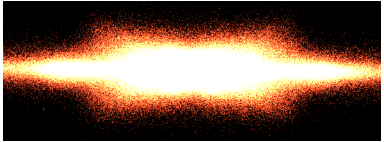
$A_{\rm pea}$: X/peanut amplitude
$R_{\rm pea}$: peanut location
$w_{\rm pea}$: peanut width

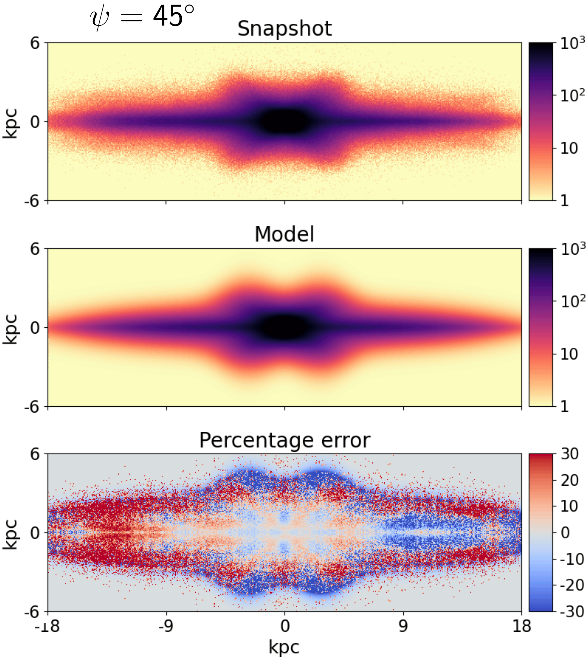
First application: edge-on projections

face-on view



edge-on view, $\psi = 0$



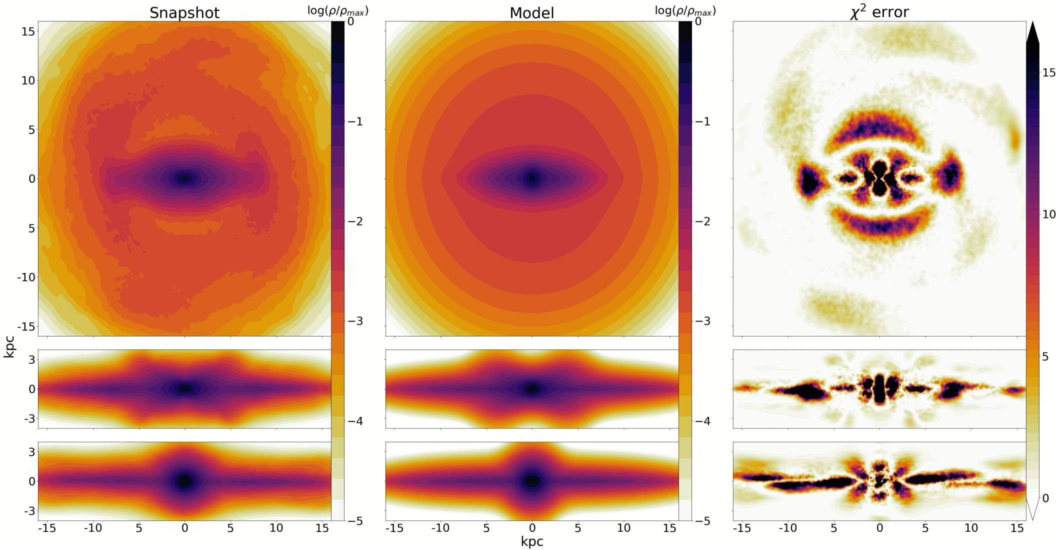


[Dattathri+ 2024]

First application: edge-on projections

The fitted model qualitatively recovers the 3d density profile, though not without some defects

$\psi = 45^\circ$



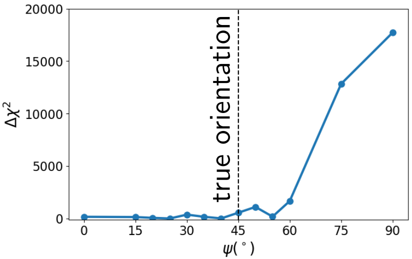
[Dattathri+ 2024]

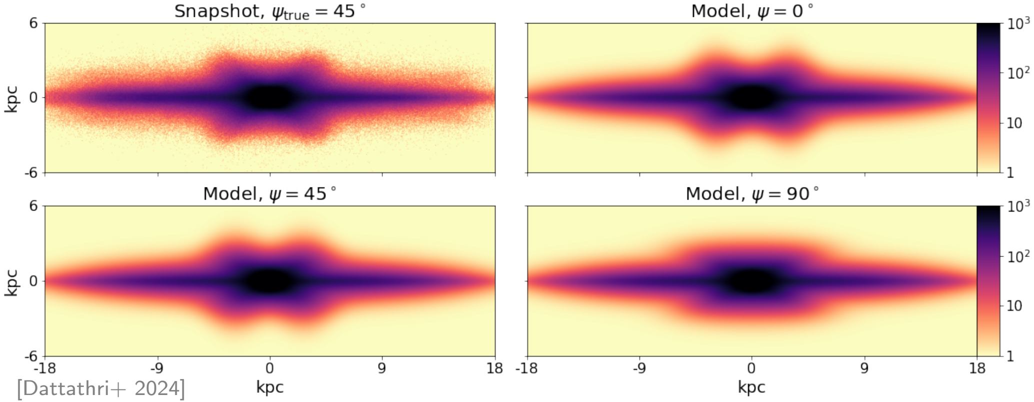
Degeneracies in determining bar orientation

It is impossible to distinguish a rotated bar $(0 < \psi < i_{\rm max} \lesssim 90^\circ)$ from a shorter bar viewed at $\psi = 0^\circ$ just from photometry.

(It might be easier at lower inclinations $i < 90^\circ$).

Kinematics / dynamical modelling should help?




Snapshot, $\psi_{\rm true} = 45^\circ$

Model, $\psi = 0^\circ$

Model, $\psi = 45^\circ$

Model, $\psi = 90^\circ$

[Dattathri+ 2024]

Modelling approaches for barred galaxies

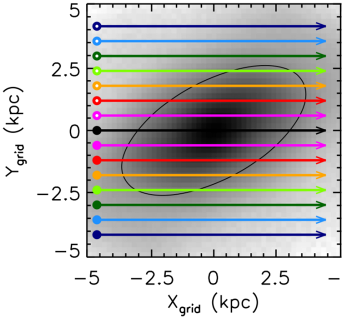
Challenges: triaxial geometry, chaotic regions in phase space

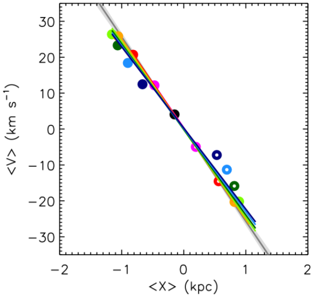
Goals:	Ω	Φ
Jeans modelling	$-$	$-$
Distribution functions, e.g., $f(\mathbf{J})$	?	?
Tremaine–Weinberg	$\pm$	$-$
Orbital response models	$+$	$+$
Guided N-body simulations (made-to-measure)	$+$	$+$
Schwarzschild orbit-superposition modelling	$+$	$+$

Measurement of the pattern speed: Tremaine & Weinberg (1984)

$$
\begin{aligned}
\Omega_p \sin i &= \frac{\langle V_{\rm los}\rangle}{\langle X\rangle} \\
\langle V_{\rm los}\rangle(Y) &\equiv \frac{\int_{-\infty}^{\infty} V_{\rm los}(X,Y)\,\Sigma(X,Y)\,\mathrm{d}X}{\int_{-\infty}^{\infty} \Sigma(X,Y)\,\mathrm{d}X} \\
\langle X\rangle(Y) &\equiv \frac{\int_{-\infty}^{\infty} X\,\Sigma(X,Y)\,\mathrm{d}X}{\int_{-\infty}^{\infty} \Sigma(X,Y)\,\mathrm{d}X}
\end{aligned}
$$

- \+ Simple to implement
- − Works only in a limited range of inclinations $15^\circ \lesssim i \lesssim 50\text{–}70^\circ$ [Zou+ 2019; Borodina+ 2023]
- − Need to integrate to $\pm\infty$ (see Dehnen+ 2023 for improvement)





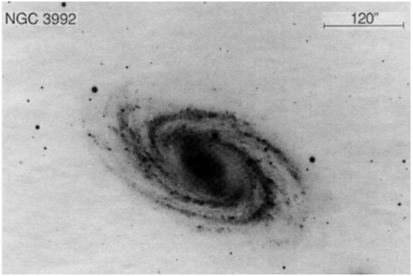
[Zou+ 2019]

Modelling approaches for barred galaxies: response models

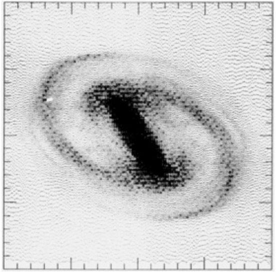
[Contopoulos & Grosbøl 1986, 1988; Patsis+ 1991; Kaufmann & Contopoulos 1996]

2d response models:

- assume parameters for potential, pattern speed, etc.
- construct the network of periodic orbits
- populate nearby orbits and compute their surface density
- compare morphological features with observations
- vary the parameters until a good match is found



observed galaxy



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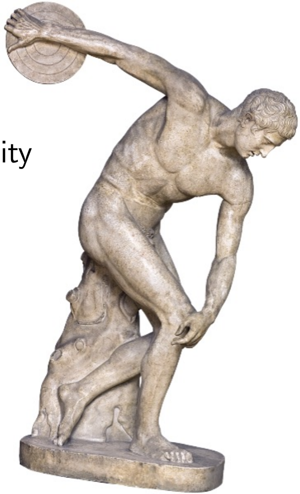

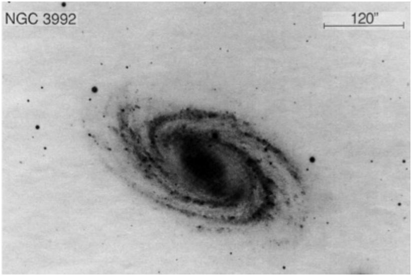

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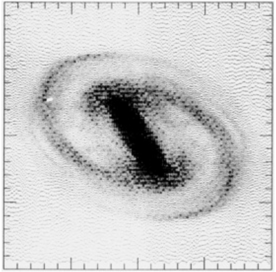


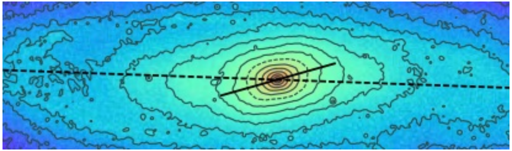
response model

Modelling approaches for barred galaxies: made-to-measure

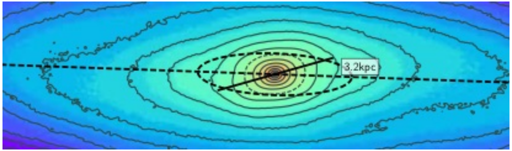
Introduced by Syer & Tremaine 1996,
grown up and flourished in Ortwin Gerhard's group
[Bissantz+ 2004, de Lorentzi+ 2007, Portail+ 2015; Blaña+ 2019];
several other implementations exist [Dehnen 2009;
Long & Mao 2012; Hunt & Kawata 2013; Malvido & Sellwood 2015].

Idea: evolve an N-body model while continuously adjusting particle weights to match the observables (density and kinematics).

Has been applied to the Milky Way bar [Portail+ 2017] and to a few external galaxies.



observed galaxy (M31)



M2M model [Blaña+ 2019]

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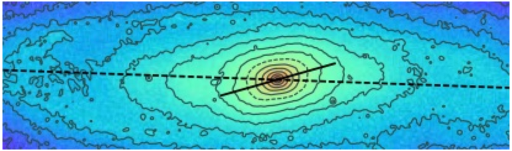

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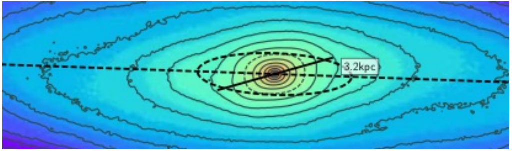

M2M model [Blaña+ 2019]

Modelling approaches for barred galaxies: orbit superposition

Introduced by Schwarzschild (1979) as a practical approach for constructing dynamically self-consistent triaxial models with prescribed $\rho(\mathbf{x}) \Leftrightarrow \Phi(\mathbf{x})$.

integrals of motion

To invert the equation $\rho(\mathbf{x}) = \iiint f\left(\mathcal{I}\left[\mathbf{x},\mathbf{v} \mid \Phi\right]\right)\, d^3\mathbf{v}$,

discretize both the density profile and the distribution function:

$\rho(\mathbf{x}) \implies$ cells of a spatial grid; mass of each cell is $M_c = \iiint\limits_{\mathbf{x}\in V_c} \rho(\mathbf{x})\, d^3x$;

$f(\mathcal{I}) \implies$ collection of orbits with unknown weights [to be determined]:

$$f(\mathcal{I}) = \sum_{k=1}^{N_\mathrm{orb}} w_k\ \delta(\mathcal{I} - \mathcal{I}_k)$$

each orbit is a delta-function in the space of integrals of motion

adjustable weight of each orbit

Modelling approaches for barred galaxies: orbit superposition


Max Ernst – Ubu Imperator

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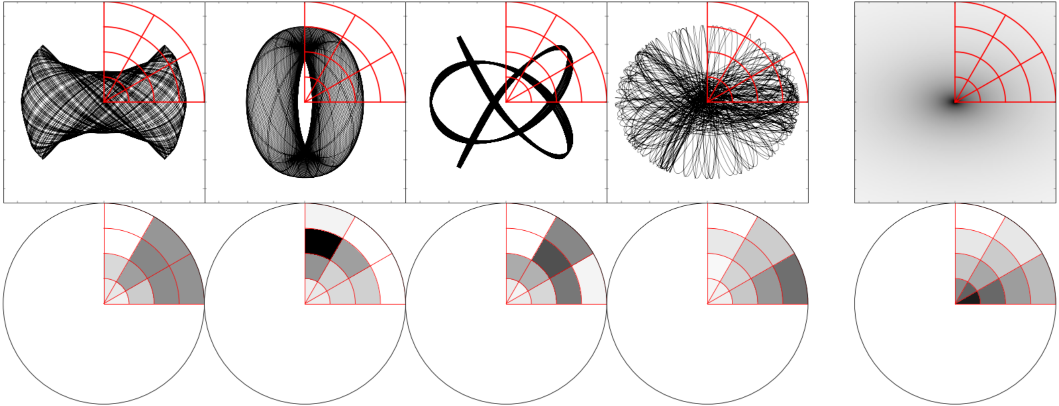
Schwarzschild's orbit-superposition method: self-consistency



orbits in the model

target density

discretized orbit density

(fraction of time t_{kc} that k-th orbit spends in c-th cell)

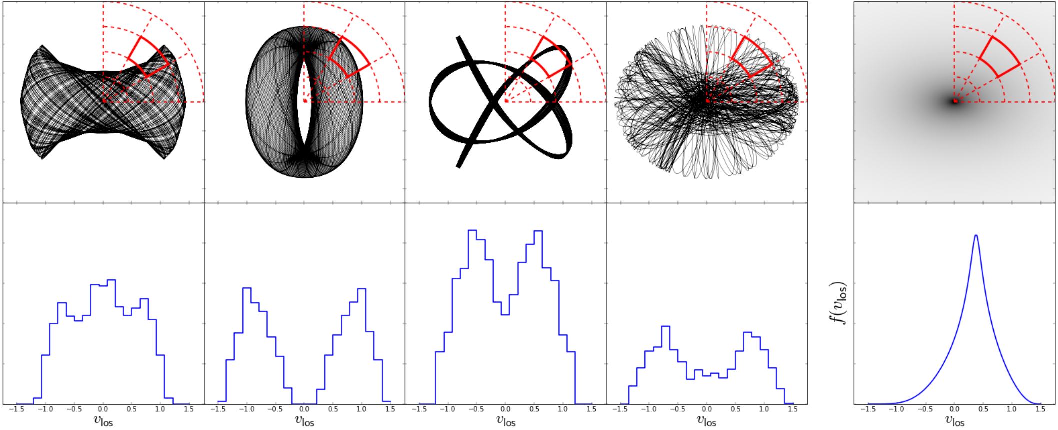
discretized density

(mass M_c in grid cells)

For each c-th cell we require $\sum_k w_k \, t_{kc} = M_c$, where $w_k \geq 0$ is orbit weight

Schwarzschild's orbit-superposition method: kinematics

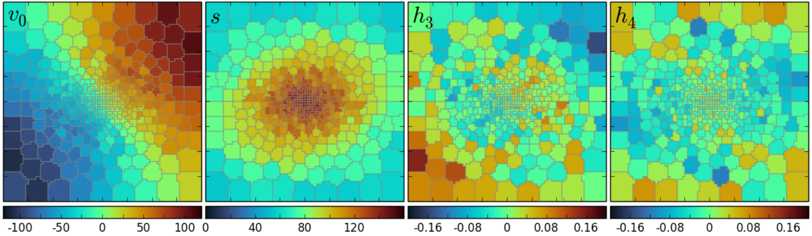
orbits in the model

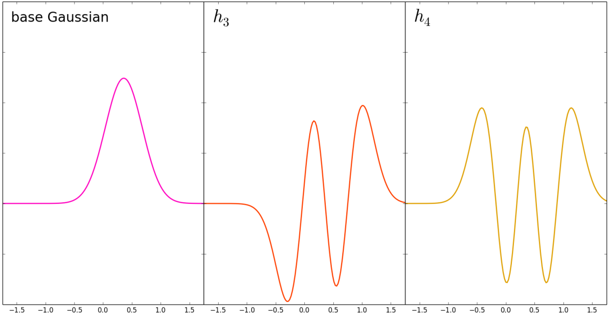


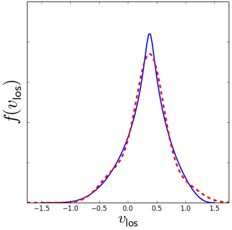
orbit LOSVDs

target LOSVD

Schwarzschild's orbit-superposition method: kinematics



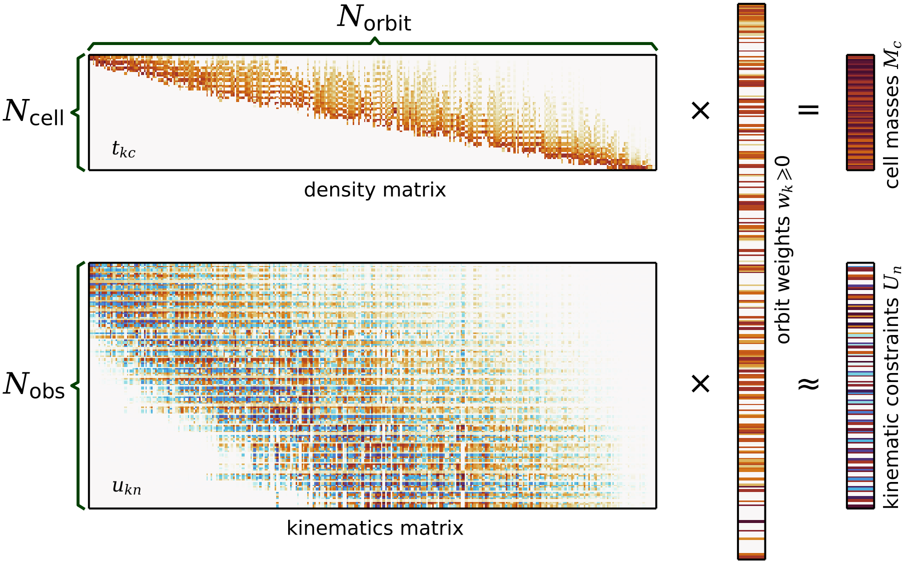




Gauss–Hermite parametrization of LOSVDs [van der Marel & Franx 1993; Gerhard 1993]

Schwarzschild's orbit-superposition method: fitting procedure

Solve the linear system with non-negativity constraints on the solution vector $w_k \geq 0$ (linear or non-linear optimization problem)



Schwarzschild's orbit-superposition method: fitting procedure

- Assume some potential $\Phi(\mathbf{x})$
  (e.g., from the deprojected luminosity profile plus parametric DM halo or SMBH)
- Construct the orbit library in this potential:
  for each k-th orbit, store its contribution to the discretized density profile
  $t_{kc}, c = 1..N_{\mathrm{cell}}$ and to the kinematic observables $u_{kn}, n = 1..N_{\mathrm{obs}}$
- Solve the constrained optimization problem to find orbit weights w_k:

$$\text{minimize } \chi^2 + \mathcal{S} \equiv \sum_{n=1}^{N_{\mathrm{obs}}} \left(\frac{\sum_{k=1}^{N_{\mathrm{orb}}} w_k \, u_{kn} - U_n}{\delta U_n} \right)^2 + \mathcal{S}(\{w_k\})$$

regularization term

observational constraints

their uncertainties

$$\text{subject to } w_k \ge 0, \quad k = 1..N_{\mathrm{orb}},$$

$$\sum_{k=1}^{N_{\mathrm{orb}}} w_k \, t_{kc} = M_c, \quad c = 1..N_{\mathrm{cell}}$$

density constraints (cell masses)

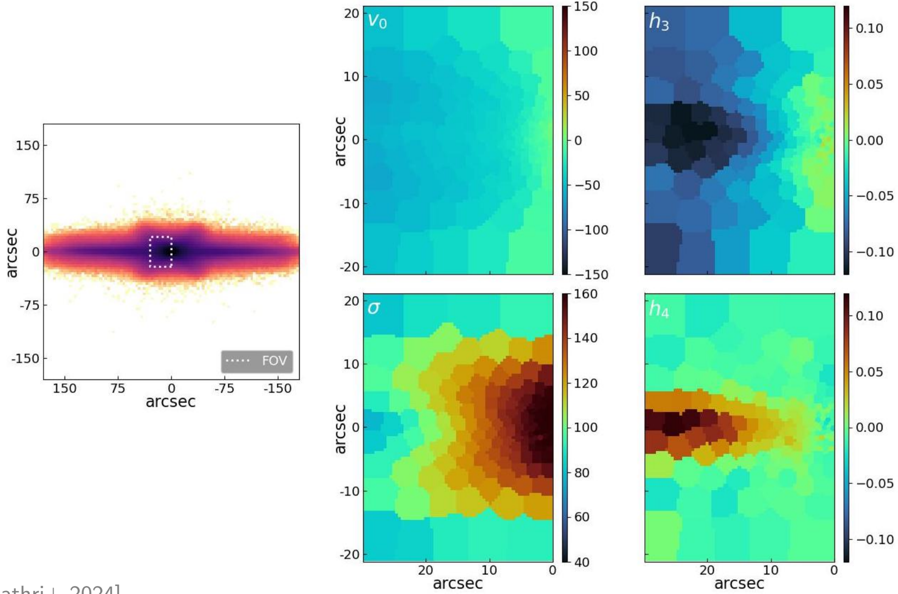
- Repeat for different choices of potential and find the one that has lowest χ^2

Schwarzschild's orbit-superposition method: implementations

- theoretical studies in triaxial geometry: Schwarzschild 1979, 1993; Pfenniger 1984; Statler 1987; Merritt & Fridman 1996; Siopis & Kandrup 2000; Vasiliev 2013
- spherical codes: Richstone & Tremaine 1984; Rix+ 1997; Jalali & Tremaine 2010; Breddels & Helmi 2013; Kowalczyk+ 2017
- axisymmetric: "Leiden" [van der Marel, Cretton, Cappellari, . . . – since 1998]
- axisymmetric: "Nukers" [Gebhardt, Richstone, Kormendy, . . . – since 2000]
- axisymmetric: "MasMod" [Valluri, Merritt, Emsellem – since 2004]
- triaxial/Milky Way bar: Zhao, Wang, Mao 1996, 2012
- triaxial: van den Bosch, van de Ven . . . – since 2008 ⇒ "Dynamite"
- triaxial: "SMART" [Neureiter+ 2021]
- triaxial: "Forstand" [Vasiliev & Athanassoula 2015; Vasiliev & Valluri 2020]

Schwarzchild modelling of deprojected bars

MUSE-like kinematic maps ($1'$ FoV) of a Milky Way-like galaxy at $D = 20$ Mpc

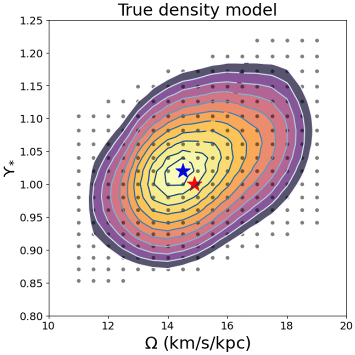


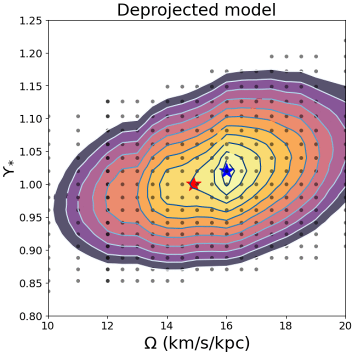
[Dattathri+ 2024]

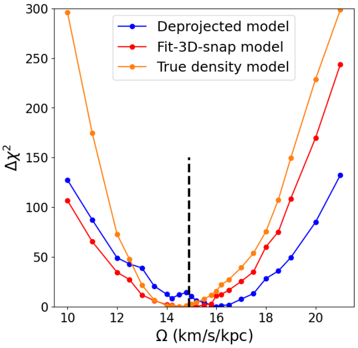
Recovery of bar pattern speed

Ω is recovered almost perfectly if the true 3d density is used, or to within 10% if the deprojected density is used.

This is for the most challenging edge-on orientation, where the Tremaine–Weinberg method is not applicable!


True density model

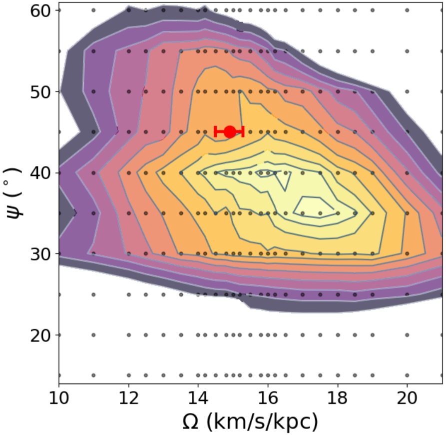

Deprojected model

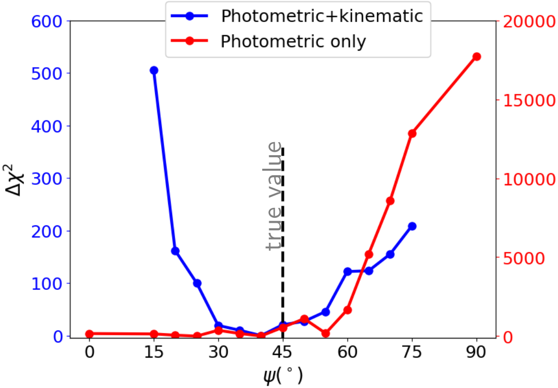

$\psi = \psi_{\text{true}} = 45°$

[Dattathri+ 2024]

Recovery of bar orientation

Bar orientation is also constrained much better than from pure photometry



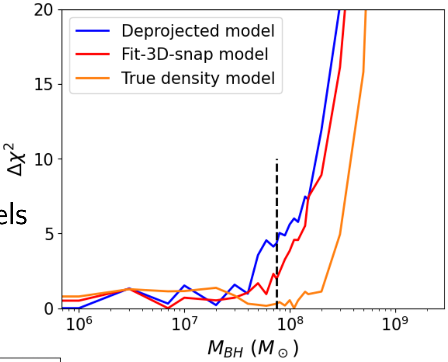


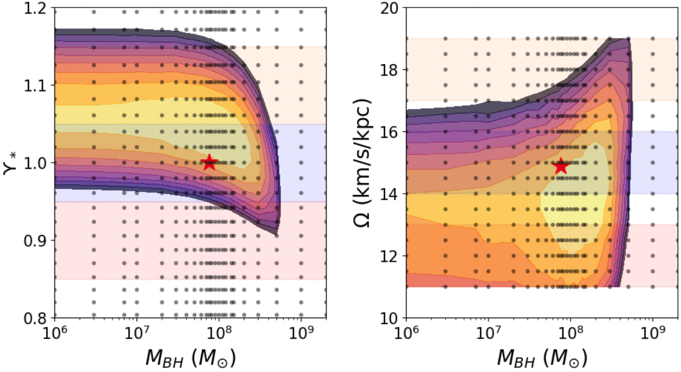
[Dattathri+ 2024]

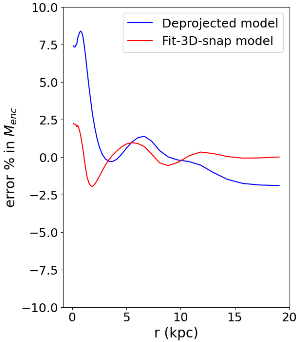
Recovery of black hole mass

Central supermassive black hole

- does not destroy the bar [Wheeler+ 2023]
- has only an upper limit on $M_\bullet$ in these models
- is very sensitive to the accuracy of reconstruction of enclosed stellar mass

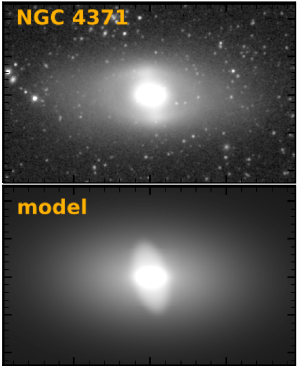


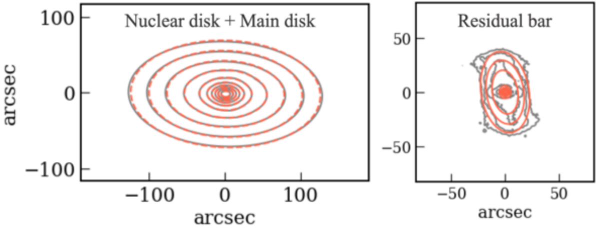


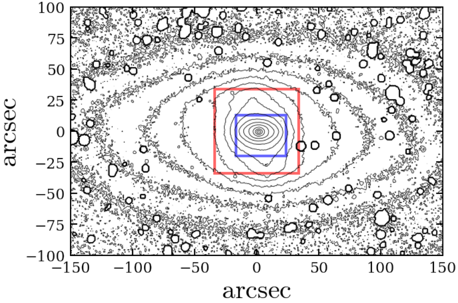


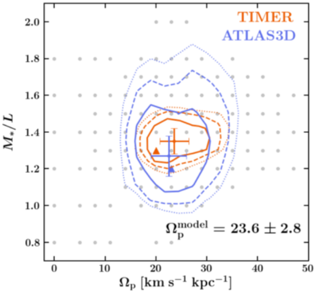
Another implementation of Schwarzschild's method for bars

by Tahmasebzadeh+ 2021, 2022, based on the triaxial code DYNAMITE;
use MGE for deprojection, separately for the main disc and the bar









[Tahmasebzadeh+ 2024]

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- Photometric bar deprojection:
  possible with IMFIT, but has some degeneracies.
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  recovers pattern speed, orientation and stellar mass.

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