Dynamical models of the Milky Way nuclear star cluster

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Dynamical modelling overview: input data

(individual stars)

positions, magnitudes

proper motions $\boldsymbol{\mu}_i$

line-of-sight velocities $V_{\mathrm{LoS},i}$

(unresolved)

surface brightness profile

integrated-light kinematics
(line-of-sight only)

$\mathbf{x}, \mathbf{v}$ – intrinsic (model) coordinates; $\mathbf{X}, \mathbf{V} \equiv \{V_{\mathrm{LoS}}, \mu_X, \mu_Y\}$ – observed data
indices: i – stars;

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$\Sigma(\mathbf{X})$

deprojection $\Downarrow$

luminosity density $j(\mathbf{x})$

mass-to-light ratio $\Downarrow$

mass density $\rho_\star(\mathbf{x})$

Poisson eq $\Downarrow$

$$\Phi(\mathbf{x}) = -\frac{GM_\bullet}{r} + \Phi_\star(\mathbf{x}) + \dots$$

photometry

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photometry

(optional) spatial binning

velocity moments $\overline{\mathbf{V}}^{(b)}$, $\boldsymbol{\sigma}^{(b)}$ at $\mathbf{X}^{(b)}$

velocity distribution functions $\mathfrak{f}^{(b)}(\mathbf{V})$, often parameterised by Gauss–Hermite moments $v_0, \varsigma, h_3, h_4, \dots$

kinematics

$\mathbf{x}, \mathbf{v}$ – intrinsic (model) coordinates; $\mathbf{X}, \mathbf{V} \equiv \{V_{\mathrm{LoS}}, \mu_X, \mu_Y\}$ – observed data
indices: i – stars; b – bins;

Dynamical modelling overview: methods

input kinematic data

(individual stars)

$\mathbf{X}_i, \mathbf{V}_i$

(binned)

$\overline{\mathbf{V}}^{(b)}$, $\boldsymbol{\sigma}^{(b)}$ or

Gauss–Hermite moments
$v_0, \varsigma, h_3, h_4, \dots,$ or

full VDF $\mathfrak{f}^{(b)}(\mathbf{V})$

indices: i – stars; b – bins;

Dynamical modelling overview: methods

orbit fitting of multi-epoch observations

[Genzel+ 2003+; Ghez+ 2003+; GRAVITY 2018+]

input kinematic data

(individual stars)

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(binned)

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Dynamical modelling overview: methods

input kinematic data

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orbit fitting of multi-epoch observations
[Genzel+ 2003+; Ghez+ 2003+; GRAVITY 2018+]

Jeans equations:
$\rho_\star(\mathbf{x}),\ \Phi(\mathbf{x}) \ \Rightarrow\ \overline{\mathbf{V}}^{\rm mod}(\mathbf{X}),\ \boldsymbol{\sigma}^{\rm mod}(\mathbf{X})$
compare with binned $\overline{\mathbf{V}}^{(b)}$, $\boldsymbol{\sigma}^{(b)}$ at $\mathbf{X}^{(b)}$
[Schödel+ 2009; Do+ 2013; Feldmeier+2014; Fritz+ 2016]

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or "discrete Jeans" – assume a Gaussian DF

$\mathfrak{f}(\mathbf{V}_i) = \mathcal{N}\big(\mathbf{V}_i;\ \overline{\mathbf{V}}^{\rm mod}, \boldsymbol{\sigma}^{mod}$ at $\mathbf{X}_i\big)$ [Do+ 2020]

indices: i – stars; b – bins;

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DF-based models with parameters p:

evaluate $f\left(\mathcal{I}(\mathbf{x}, \mathbf{v};\ \Phi),\ \mathbf{p}\right)$ at each star;

integrals of motion $\mathcal{I}$ depend on Φ

[Chatzopoulos+ 2015; Magorrian+ 2019; this study]

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Schwarzschild orbit-superposition models

$f(\mathcal{I}) = \sum_{k=1}^{N_{\mathrm{orbits}}} w_k\, \delta\big(\mathcal{I} - \mathcal{I}_k[\Phi]\big)$

compare model VDFs $\mathfrak{f}^{(b)}$ or GH moments

[Feldmeier+ 2017]

indices: i – stars; b – bins; k – orbits

Distribution functions

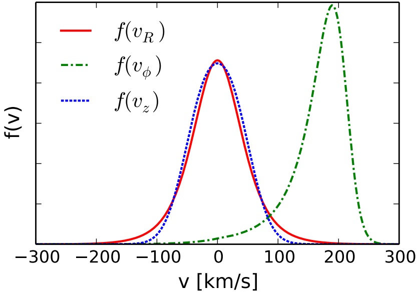
DF $f(\mathbf{x},\mathbf{v})$ offers a complete description of the stellar population:

- density $\rho(\mathbf{x}) = \iiint f(\mathbf{x},\mathbf{v})\,\mathrm{d}^3 v$,
- mean velocity $\overline{\mathbf{v}}(\mathbf{x}) = \dfrac{1}{\rho(\mathbf{x})} \iiint \mathbf{v}\, f(\mathbf{x},\mathbf{v})\,\mathrm{d}^3 v$,
- second moment of velocity $\overline{v_{ij}^2}(\mathbf{x}) = \dfrac{1}{\rho(\mathbf{x})} \iiint v_i\, v_j\, f(\mathbf{x},\mathbf{v})\,\mathrm{d}^3 v$,

  velocity dispersion tensor $\sigma_{ij}^2 \equiv \overline{v_{ij}^2} - \overline{v_i}\,\overline{v_j}$,
- more generally, 1d velocity distribution at a given point

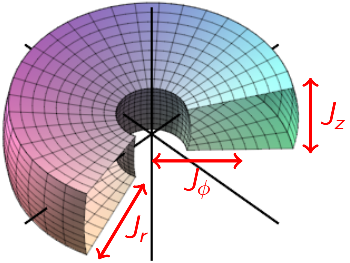
  $\mathfrak{f}(v_1;\,\mathbf{x}) = \dfrac{1}{\rho(\mathbf{x})} \iint f(\mathbf{x},\mathbf{v})\,\mathrm{d}v_2\,\mathrm{d}v_3$

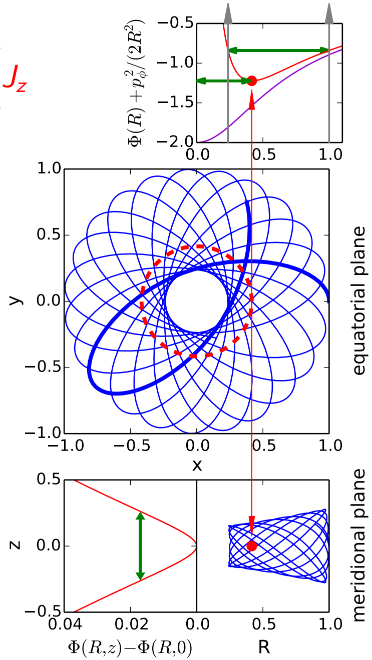
  (note that it may be rather non-Gaussian!).



Jeans' theorem: in a steady state, DF must be a function of integrals of motion $f\left(\mathcal{I}(\mathbf{x},\mathbf{v};\ \Phi)\right)$, for which it may be convenient to use actions $\mathbf{J}$.

Action–angle variables





Orbits in axisymmetric potentials look like "rectangular tori" with three parameters defining the shape:

angular momentum $J_\phi \equiv L_z = R_g\, v_{circ}(R_g)$ determines the overall size of the orbit ("guiding radius" R_g);

radial action J_R defines the extent of radial oscillations;

vertical action J_z does the same for vertical oscillations.

Actions are computed in the Stäckel approximation [Binney 2012], as implemented in AGAMA [Vasiliev 2019].

Iterative construction of self-consistent dynamical models

A model can be fully specified by its DF $f(\mathcal{I})$, and the density $\rho(\mathbf{x})$ and potential $\Phi(\mathbf{x})$ follow from it (e.g. a King model).

However, to compute $\rho(\mathbf{x}) = \iiint f\big(\mathcal{I}(\mathbf{x}, \mathbf{v};\ \Phi)\big)\ \mathrm{d}^3\mathbf{v}$, one needs to know the potential, which is, in turn, linked to density by the Poisson equation $\nabla^2\Phi = 4\pi G\rho$.

Thus we have a circular dependency, which is resolved using an iterative approach [Kuijken & Dubinski 1995; Widrow+ 2005; Binney 2014; Piffl+ 2015; Binney & Vasiliev 2023, 2024].

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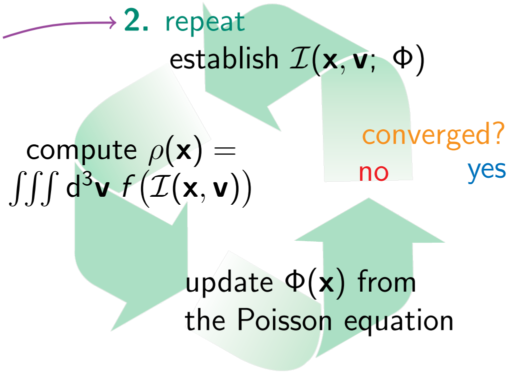
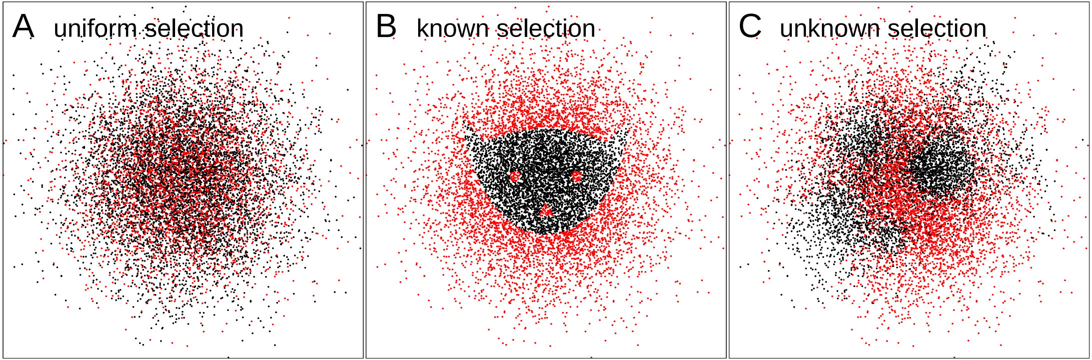
1. assume $f(\mathcal{I})$ and an initial guess for Φ



3. enjoy!

Likelihood of a DF-based model

[spatial] selection function $S(\mathbf{x})$ – fraction of stars included in the kinematic sample



$$\ln \mathcal{L} = \sum_{i=1}^{N_{\text{stars}}} \ln f(\mathbf{x}_i, \mathbf{v}_i)$$

$$\ln \mathcal{L} = \sum_{i=1}^{N_{\text{stars}}} \ln \frac{f(\mathbf{x}_i, \mathbf{v}_i)}{\mathcal{N}},$$

$$\mathcal{N} \equiv \iiint d^3\mathbf{x} \iiint d^3\mathbf{v}\, f(\mathbf{x}, \mathbf{v})\, S(\mathbf{x})$$

(normalization factor)

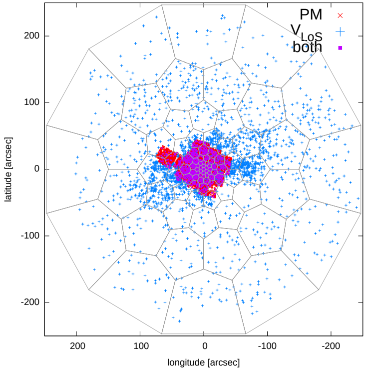
$$\ln \mathcal{L} = \sum_{i=1}^{N_{\text{stars}}} \ln \mathfrak{f}(\mathbf{v}_i \mid \mathbf{x}_i),$$

$$\mathfrak{f}(\mathbf{v} \mid \mathbf{x}) \equiv \frac{f(\mathbf{x}, \mathbf{v})}{\iiint d^3\mathbf{v}\, f(\mathbf{x}, \mathbf{v})} = \frac{f(\mathbf{x}, \mathbf{v})}{\rho(\mathbf{x})}$$

(conditional velocity distribution)

Modelling workflow

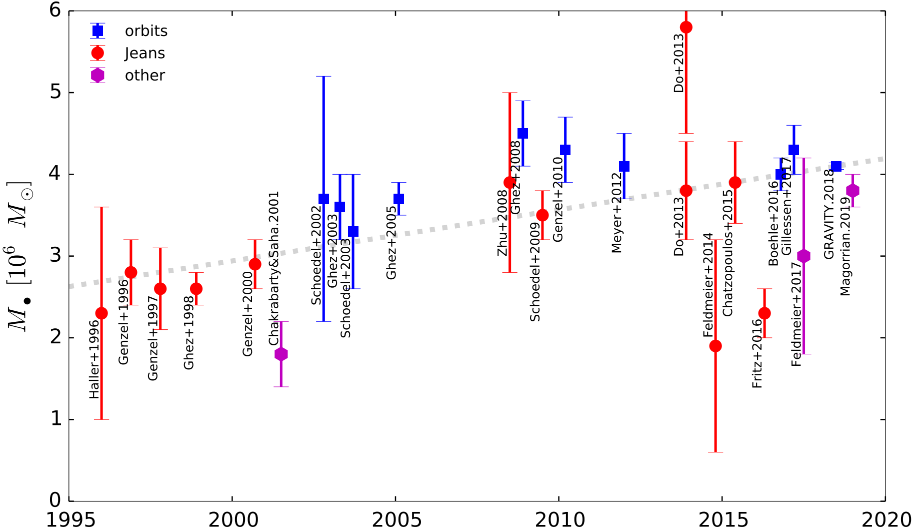
- Adopt parameters **p** describing the NSC DF (e.g., mass, inner/outer slope, anisotropy, rotation) and BH mass $M_\bullet$; the NSD DF is kept fixed.
- Construct the self-consistent model .
- Compute the 1d velocity distribution functions (VDFs) $\mathfrak{f}^{(b)}(v_{x,y,z}\,|\,\mathbf{x}^{(b)})$ in ~ 100 spatial bins.
- Evaluate the likelihood $\mathcal{L}$ of the observed dataset given the model VDFs (sum of NSC and NSD).
- Repeat with a different choice of parameters **p** in the MCMC loop.



Note that there is no binning of observational data!
bins are only used to construct spatially varying model VDFs,
and each star's contribution to the likelihood is summed up separately.

We also do not input any information about the density profile (although be could..)

Black hole mass growth



(figure originally presented at the "Dense stellar systems" workshop in Heidelberg, Novermber 2018)

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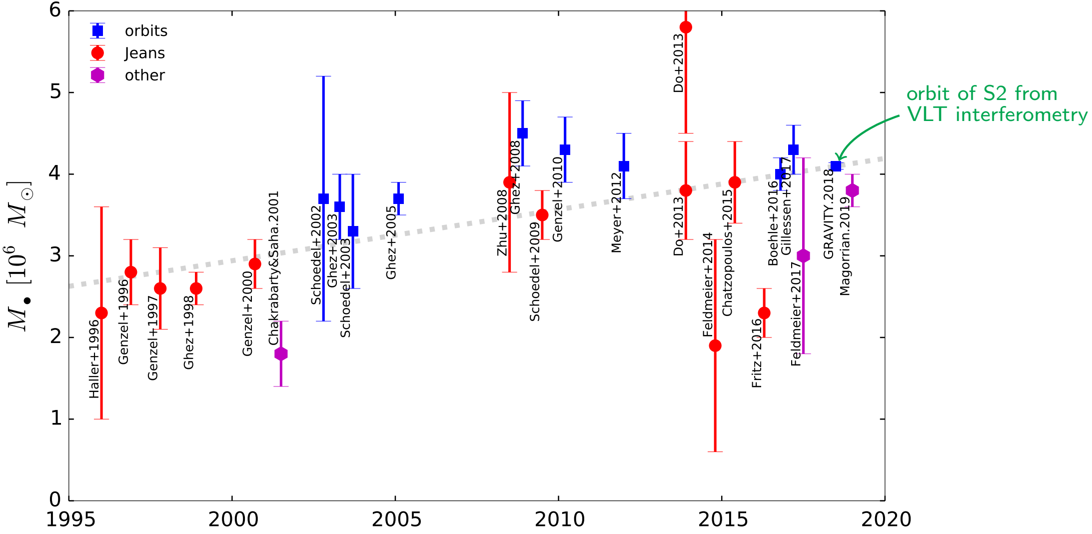

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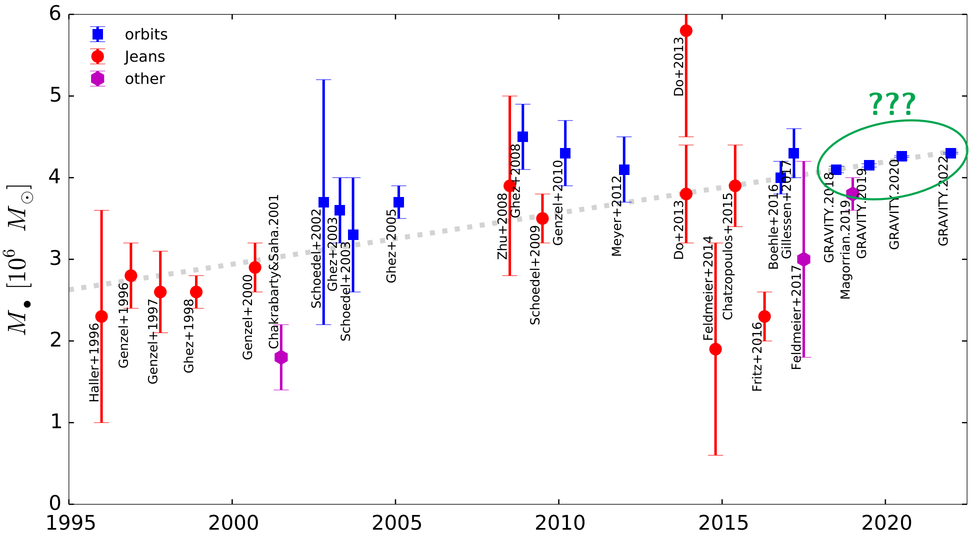

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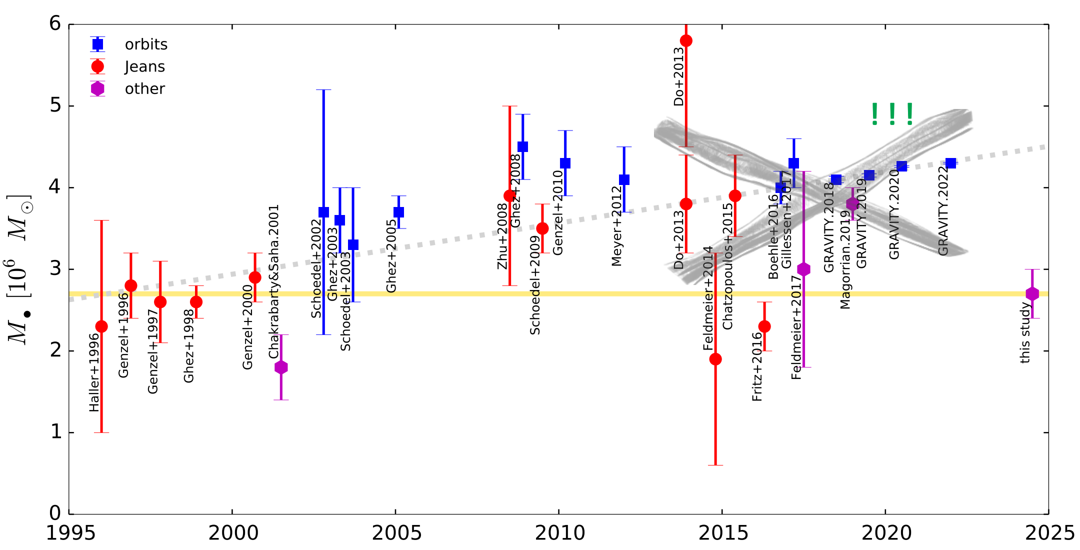
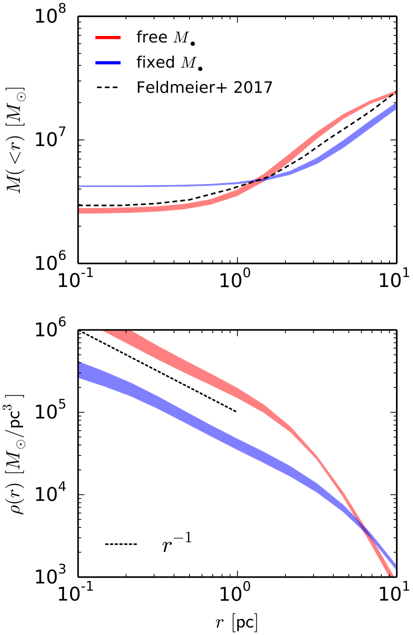


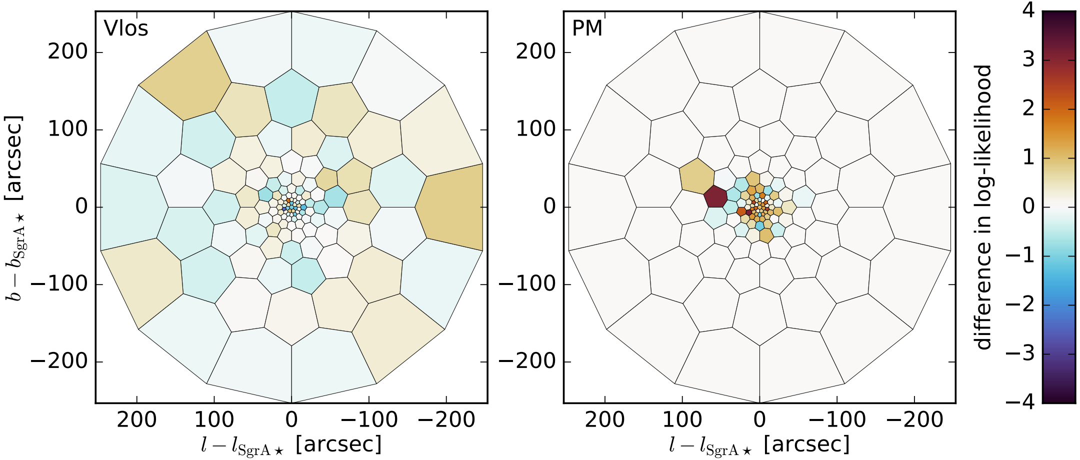
What happens?



- Enclosed mass in the radial range 1–2 pc is the same in models with free and fixed (true) $M_\bullet$, but the radial profiles differ.
- Stellar density profile has a similar shape (a mild cusp $\rho \propto r^{-1}$) in both cases, somewhat shallower than inferred from the photometric observations [Gallego-Cano+ 2018, Schödel+ 2018]. Note that it was not explicitly put into the model!

Why a lower $M_\bullet$ is preferred by models?

difference in log-likelihood between models with free and fixed $M_\bullet$



- Most of the difference comes from PM;
- v_{LoS} data alone cannot constrain the SMBH mass well, even though they cover a much larger spatial region.

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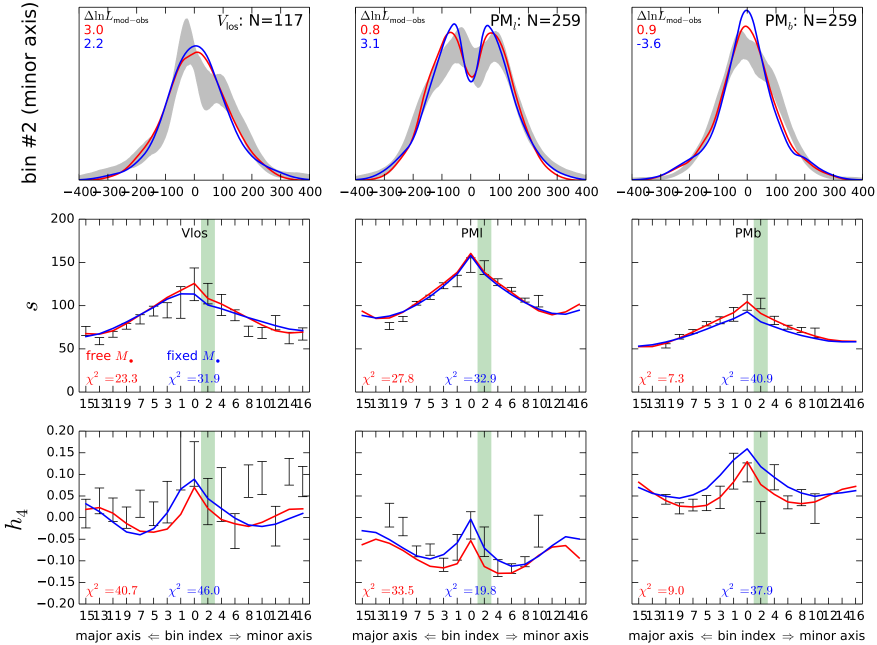

velocity distributions (only one bin shown)

free $M_\bullet$

fixed (true) $M_\bullet$

observations

Gauss–Hermite moments of velocity distributions in all bins

models with free $M_\bullet$ better fit the shape of the VDF!

Discussion

Testing the DF fitting method on mock data: $M_{\bullet,\text{best-fit}} \simeq (3.9{\pm}0.2){\times}10^6\,M_\odot$, close to the actual value.

Possible culprits:

- some features in the data? unlikely, tried two independent PM datasets (Schödel+ 2009; Fritz+ 2016) with no significant difference in results.
- biases from model mismatch (limitations of the functional form of the DF) (e.g., it becomes necessarily isotropic in the limit of a cusp slope $r^{-3/2}$)

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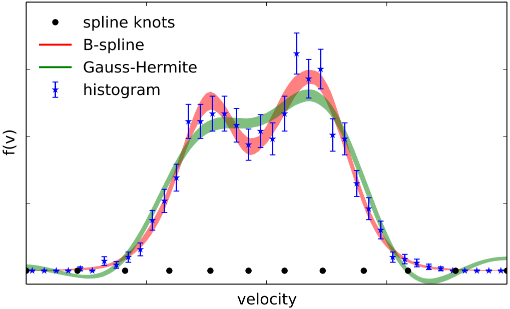
OTOH a *dynamical* model with ~ 10 free parameters in the DF fits the VDFs in ~ 20 bins just as well as *empirical* B-spline fits to VDFs with 10 free parameters per bin!



Summary

- DF-based models offer a more detailed description of the NSC + NSD system than just the first two velocity moments;
- The bias in the recovery of SMBH mass is puzzling, possibly owes to limitations of the adopted functional form of the DF;
- A caveat for measuring IMBH masses in globular clusters?

Summary

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- The bias in the recovery of SMBH mass is puzzling, possibly owes to limitations of the adopted functional form of the DF;
- A caveat for measuring IMBH masses in globular clusters?

"I have all data and tools,
but what to do with the results??"