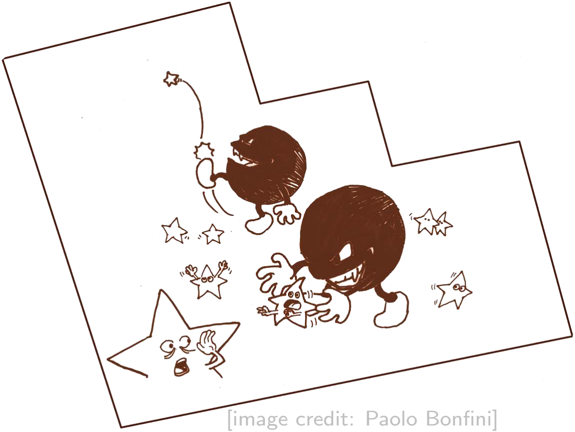
# Evolution of binary supermassive black holes and the mythical final-parsec problem



[image credit: Paolo Bonfini]

*WFPC2 captures a SMBH binary kicking stars out of the bulge*

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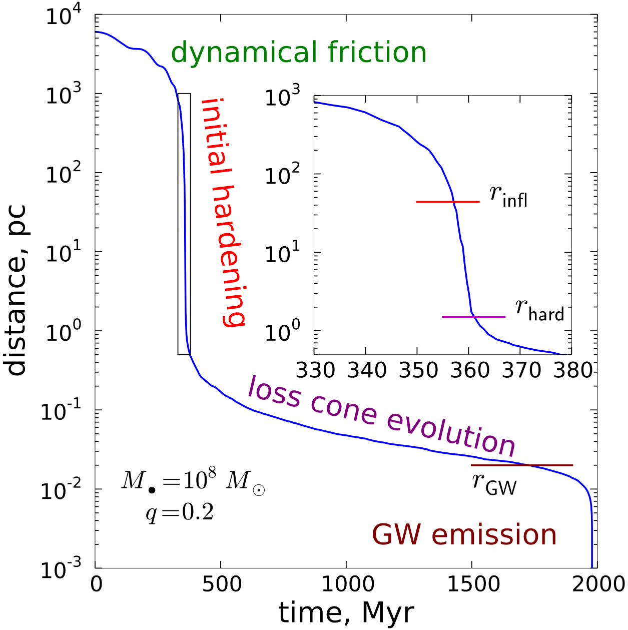
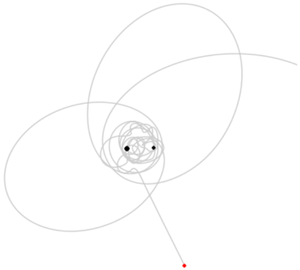
# Evolutionary stages of binary supermassive black holes



# Gravitational slingshot and binary hardening



Three-body scattering:

a star passing near the binary is ejected with a typical velocity

$v_{\rm ej} \sim \sqrt{\frac{m_1 m_2}{(m_1+m_2)^2}}\, v_{\rm bin} \gg \sigma.$

These stars carry away energy and angular momentum, so that the binary semimajor axis $a$ decreases:

$$\frac{d}{dt}\left(\frac{1}{a}\right) \approx 16\frac{G\,\rho}{\sigma} \equiv S_{\rm full}$$

[Quinlan 1996]

Thus if the density of stars $\rho$ remains constant, the binary hardens at a constant rate. However, the reservoir of low angular momentum stars (the loss cone) may be depleted quickly $\Rightarrow$ the binary stalls at a radius $a_{\rm stall} \sim (0.1 - 0.4)a_{\rm hard}$.

## Loss cone theory

Loss cone angular momentum: $L_{\rm LC} \equiv \sqrt{2G(m_1+m_2)\,a}$.

Stars with $L < L_{\rm LC}$ are eliminated on a dynamical timescale $T_{\rm dyn}$.

The crucial parameter is the timescale for loss cone repopulation.

In the absence of other processes, the repopulation time is

$T_{\rm rep} \sim T_{\rm rel}\,\frac{L_{\rm LC}^2}{L_{\rm circ}^2}$, where $T_{\rm rel} = \frac{0.34\,\sigma^3}{G^2\, m_\star\, \rho_\star\, \ln\Lambda}$ is the relaxation time.

If $T_{\rm rep} \lesssim T_{\rm dyn}$, the loss cone is full.

However, real galaxies are in the opposite (empty loss cone) regime.

In this case the hardening rate $S \equiv \frac{d}{dt}(a^{-1}) \simeq \frac{T_{\rm dyn}}{T_{\rm rep}} S_{\rm full}$.

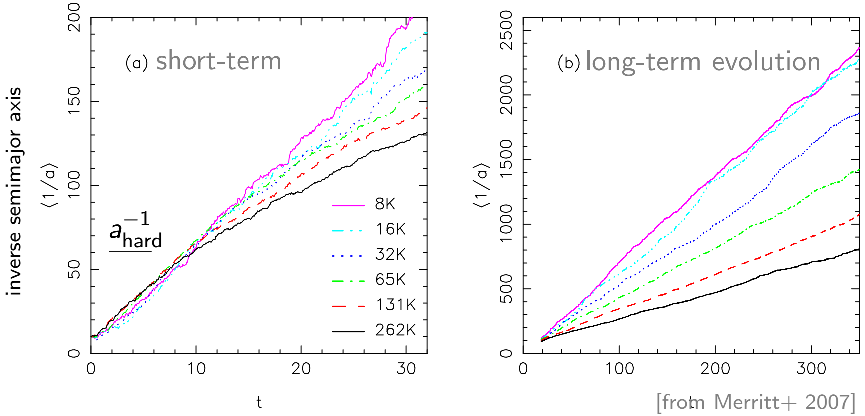
Relaxation is too slow for an efficient repopulation of the loss cone: in the absense of other processes, the binary would not merge in a Hubble time.
This is the "final-parsec problem" [Milosavljević&Merritt 2003]

# $N$-scaling in the empty loss cone regime

In galaxy-scale $N$-body simulations, the number of particles $N \lesssim 10^6$ is much less than the number of stars in the galaxy $N_\star$.

Hardening rate $S \equiv \dfrac{d}{dt}(a^{-1}) \propto T_{rel}^{-1} \propto N^{-1}$.

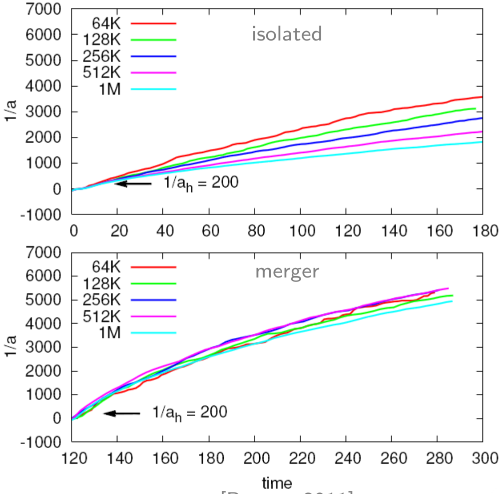
signature of empty loss cone regime



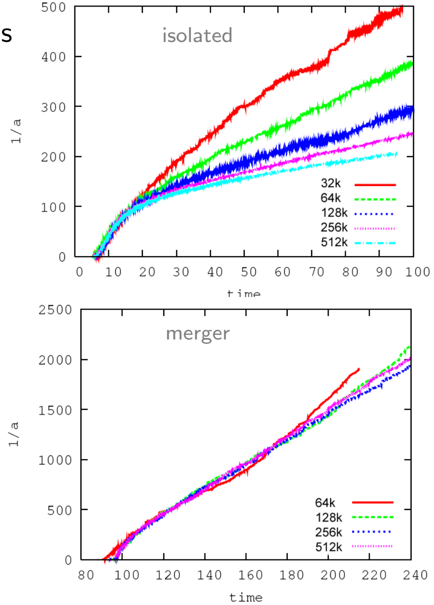

[from Merritt+ 2007]

# Merger simulations hint for a full loss cone

Hardening rates in merger simulations were found to be $N$-independent




[Preto+ 2011]



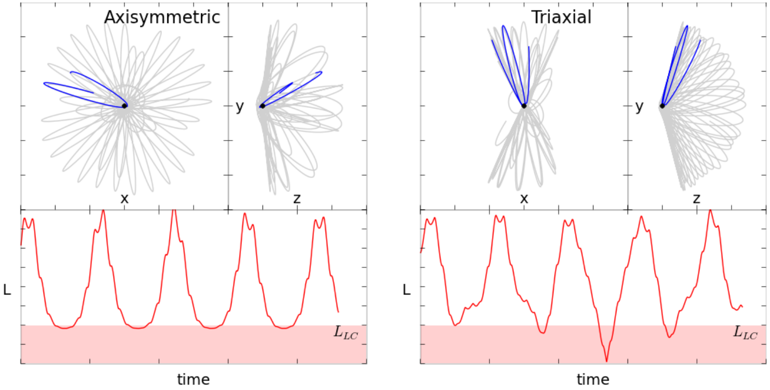

[Khan+ 2011]

# Loss cone in non-spherical stellar systems

Angular momentum $L$ of any star is not conserved, but experiences oscillations due to torques from non-spherical distribution of stars.

More stars can attain low $L$ and enter the loss cone at some point in their (collisionless) evolution, regardless of two-body relaxation.

This has led to the conclusion that the loss cone in axisymmetric and especially triaxial systems remains full.

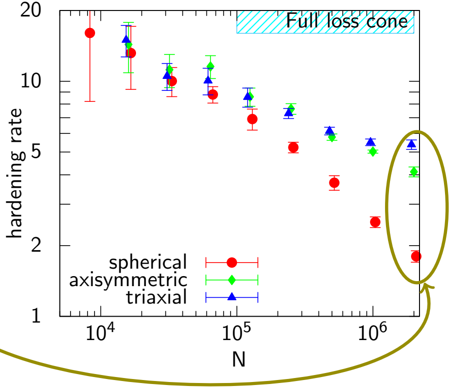


# Evolution of isolated systems in different geometries

But this can't be the whole story:

in $N$-body simulations of isolated systems with different geometry – spherical, axisymmetric and triaxial – the hardening rate still decreases with $N$ (but less strongly in non-spherical cases), and is several times lower than $S_{\mathrm{full}}$.

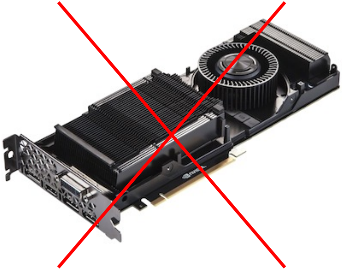
[Vasiliev, Antonini & Merritt 2014]




Is there a convergence in the limit $N \to \infty$?

# Problems with direct $N$-body simulations

- Galaxies have $N_\star \sim 10^{10-12}$, but simulations – only $N \sim 10^6$;
- Cannot simply extrapolate the hardening rate to different $N$:
  collisional relaxation scales as $N^{-1}$,
  collisionless processes are independent of $N$;
- We can't afford much higher $N$ even with the latest hardware
  (at least using direct-summation codes)



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Need a simulation method in which we may

- accurately follow fast three-body scattering events;
- track the depletion and slow repopulation of the loss cone;
- account for the change of galaxy shape and erosion of density cusp;
- adjust the relaxation rate independently of particle number
  (in particular, attain the collisionless limit by switching it off).

Sounds too good to be feasible?

# A novel simulation method

- **Suppression of relaxation:**
  use spatial and temporal smoothing and oversampling;
- **Gravitational potential:**
  spherical-harmonic expansion for $\forall$ geometry;
- **Star-binary interactions:**
  explicit tracking of energy and angular momentum exchanges in three-body scattering events;
- **Addition of relaxation:**
  local diffusion coefficients for velocity perturbations

Assumptions:

- quasi-stationary evolution, well defined center;
- hard SBH binary already formed

  [Vasiliev 2015]

# Global dynamics: smooth field method

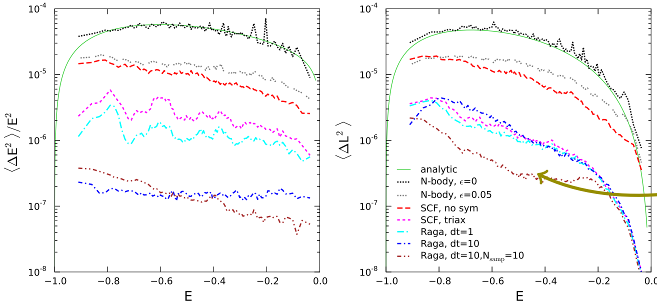
Spherical-harmonic expansion for the global stellar potential

(cf. Aarseth 1967, Hernquist&Ostriker 1992) :

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^{l} \Phi_{l,m}(r) Y_l^m(\theta,\phi);$$

$$\Phi_{l,m}(r) = -\frac{4\pi G}{2l+1} \left[ r^{-l-1} \sum_{r_i<r} m_i Y_l^m(\theta_i,\phi_i) r_i^l + r^l \sum_{r_i>r} m_i Y_l^m(\theta_i,\phi_i) r_i^{-1-l} \right]$$

Use long intervals between potential update ($\gtrsim T_{\rm dyn}$);
take many sampling points from each particle's trajectory.




Reduce discreteness noise by a couple of orders of magnitude for the same $N$

# Three-body scattering and binary evolution

- Two black holes on a Keplerian orbit;
- Test particles in time-dependent gravitational field;
- Record changes in energy and angular momentum of each particle, adjust the binary orbit parameters (semimajor axis $a$ and eccentricity $e$) using conservation laws [e.g. Sesana+ 2006,2007; Meiron&Laor 2012].
- Add gravitational-wave emission:

$$\left.\frac{d(1/a)}{dt}\right|_{\rm GW} = \frac{64}{5}\frac{G^3M_\bullet^3}{c^5a^5}\frac{q}{(1+q)^2}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}},$$

$$\left.\frac{de}{dt}\right|_{\rm GW} = -\frac{G^3M_\bullet^3}{c^5a^4}\frac{q}{(1+q)^2}\frac{e(304+121e^2)}{15(1-e^2)^{5/2}}$$

[Peters 1964].

# Collisional relaxation and the Monte Carlo method

Spitzer's (1971) formulation of Monte Carlo method in terms of local (position-dependent) velocity perturbations:

$$\Delta v_{\|} = \langle \Delta v_{\|} \rangle \Delta t + \zeta_1 \sqrt{\langle \Delta v_{\|}^2 \rangle \Delta t}\,,$$

$$\Delta v_{\perp} = \zeta_2 \sqrt{\langle \Delta v_{\perp}^2 \rangle \Delta t}\,, \qquad \zeta_1, \zeta_2 \sim \mathcal{N}(0,1)$$

Perturbations applied after each timestep of numerical orbit integration

$$v \langle \Delta v_{\|} \rangle = -\left(1 + \frac{m}{m_\star}\right) I_{1/2}\,,$$

$$\langle \Delta v_{\|}^2 \rangle = \frac{2}{3}\left(I_0 + I_{3/2}\right),$$

$$\langle \Delta v_{\perp}^2 \rangle = \frac{2}{3}\left(2I_0 + 3I_{1/2} - I_{3/2}\right),$$

$$I_0 \equiv \Gamma \int_E^0 dE' \, \boxed{f(E'),}$$

distribution function of stars

$$I_{n/2} \equiv \Gamma \int_{\Phi(r)}^E dE' \, f(E') \left(\frac{E' - \boxed{\Phi(r)}}{E - \Phi(r)}\right)^{n/2},$$

gravitational potential

$$\Gamma \equiv 16\pi^2 G^2 m_\star \ln\Lambda = 16\pi^2 G^2 M_{\rm tot} \times \boxed{(N_\star^{-1} \ln\Lambda).}$$

scalable amplitude of perturbation

# Implementations of the Monte Carlo method

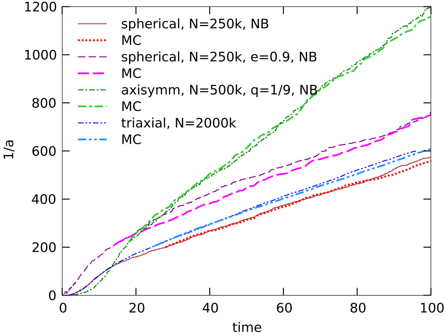
| Name | Reference | relaxation treatment | timestep | 1:1[1] | BH[2] | remarks |
|---|---|---|---|---|---|---|
| Princeton | Spitzer&Hart(1971), Spitzer&Thuan(1972) | local dif.coefs. in velocity, Maxwellian background $f(r, v)$ | $\propto T_{dyn}$ | – | – | |
| Cornell | Marchant&Shapiro (1980) | dif.coef. in $E$, $L$, self-consistent background $f(E)$ | indiv., $T_{dyn}$ | – | + | particle cloning |
| – | Hopman (2009) | same | | – | + | stellar binaries |
| Hénon | Hénon(1971) | local pairwise interaction, self-consistent bkgr. $f(r, v_\parallel, v_\perp)$ | $\propto T_{rel}$ | – | – | |
| – | Stodołkiewicz(1982) Stodołkiewicz(1986) | Hénon's | block, $T_{rel}(r)$ | – | – | mass spectrum, disc shocks binaries, stellar evolution |
| Mocca | Giersz(1998) | same | same | + | – | 3-body scattering (analyt.) |
| | Hypki&Giersz(2013) | same | same | + | – | single/binary stellar evol., few-body scattering (num.) |
| Cmc | Joshi+(2000) | same | $\propto T_{rel}(r=0)$ | + | – | partially parallelized |
| | Umbreit+(2012), Pattabiraman+(2013) | | (shared) | + | + | fewbody interaction, single/binary stellar evol., GPU |
| Me(ssy)[2] | Freitag&Benz(2002) | same | indiv.$\propto T_{rel}$ | – | + | cloning, SPH physical collis. |
| – | Sollima&Mastrobuono-Battisti(2014) | same | | – | – | realistic tidal field |
| Raga | Vasiliev(2015) | local dif.coef. in velocity, self-consistent background $f(E)$ | indiv.$\propto T_{dyn}$ | – | + | arbitrary geometry |

[1] One-to-one correspondence between particles and stars in the system

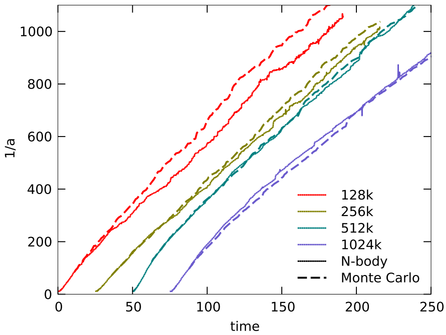
[2] Massive black hole in the center, loss-cone effects

# Calibration of Monte Carlo simulations

- Monte Carlo simulations are in quantitative agreement with direct $N$-body simulations for all combinations of parameters that we explored ($N$, mass ratio, eccentricity, geometry, ...)
- There are no free parameters in the Monte Carlo method (apart from the pre-factor $\eta \sim 0.02$ in the Coulomb logarithm $\log\Lambda = \log\eta N$).



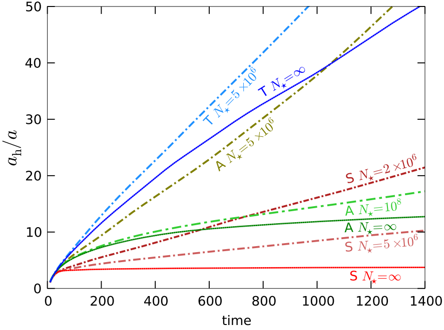

Isolated models

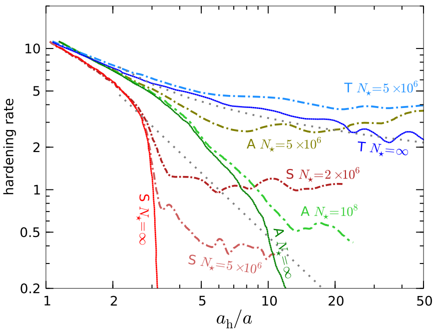



Merger simulations

# Long-term binary evolution in Monte Carlo simulations

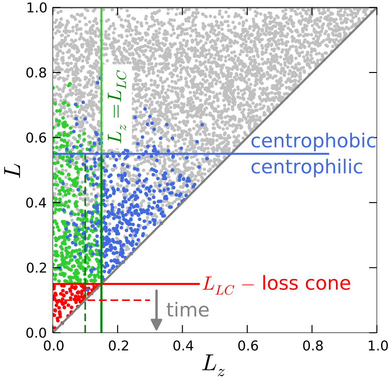
- Hardening rate decreases with time in all three geometries.
- In the absense of relaxation ($N_\star = \infty$), it drops to zero in spherical and axisymmetric cases, but stays high enough in triaxial case.
- Systems with relaxation eventually settle to a constant hardening rate at large enough time.
- There is little difference between axisymmetric and triaxial systems even for $N_\star$ as large as $5 \times 10^6$, but in the collisionless limit their evolution is qualitatively different!



# Qualitative analysis of long-term collisionless evolution

- To shrink the binary by a factor of two, one needs to eject stars with total mass $\sim M_\bullet$; thus one needs to supply a few$\times M_\bullet$ worth of stars into the loss cone over the entire evolution.
- Stars on centrophilic orbits in the extended loss region can eventually enter the loss cone; but in the axisymmetric case the volume of loss region shrinks as the binary hardens.




Particles that can arrive into the loss cone:

- not in loss region
- spherical
- axisymmetric
- triaxial

[Vasiliev, Antonini & Merritt 2015]

# Summary

- The longest evolutionary stage of a sub-parsec binary is driven by loss-cone repopulation;
- Binary black holes need few$\times 10^8 - 10^9$ years to coalesce in gas-poor galaxies;
- The "final-parsec problem" occurs in the idealized cases, but in realistic galactic mergers even minor deviations from axisymmetry are sufficient to keep the loss cone non-empty;
- Accurate treatment of this problem is difficult to achieve in conventional $N$-body simulations, but can be done with the special-purpose Monte Carlo method.

Thank you!