Galactic dynamics with AGAMA

part 2: applications

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Galaxy Modelling & Galactic centre workshop

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Chapter 1: Milky Way mass profile with 6d tracers

Problem statement:

Given a single snapshot of N 6d phase-space points $\{\mathbf{x}, \mathbf{v}\}_{i=1}^{N}$, which are assumed to be drawn from some equilibrium distribution function $f(\mathbf{x}, \mathbf{v}, \cancel{t})$ and move in some stationary gravitational potential $\Phi(\mathbf{x}, \cancel{t})$,

infer the parameters of both f and Φ.

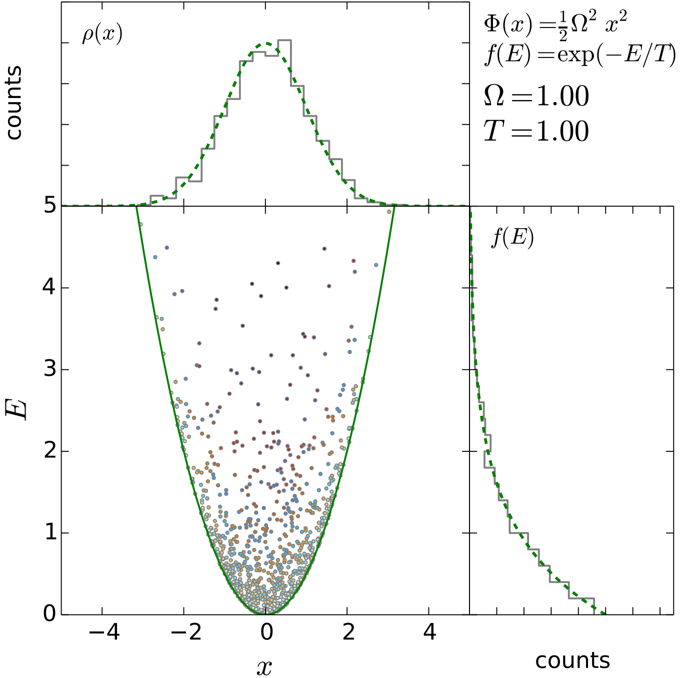
Jeans theorem tells us that in a stationary case, $f(\mathbf{x}, \mathbf{v})$ may only depend on the integrals of motion $\mathcal{I}(\mathbf{x}, \mathbf{v}\,;\ \Phi)$.

Solution:

$\boldsymbol{\alpha}, \boldsymbol{\beta} = \operatorname{argmax} \mathcal{L}$,
where the likelihood of the observed sample given the model is
$\mathcal{L} = \prod_{i=1}^{N} f\Big(\mathcal{I}\big(\mathbf{x}_i, \mathbf{v}_i\,;\ \Phi(\mathbf{x}; \boldsymbol{\alpha})\big);\ \boldsymbol{\beta}\Big)$.

But why is it possible to constrain both the DF and the potential?

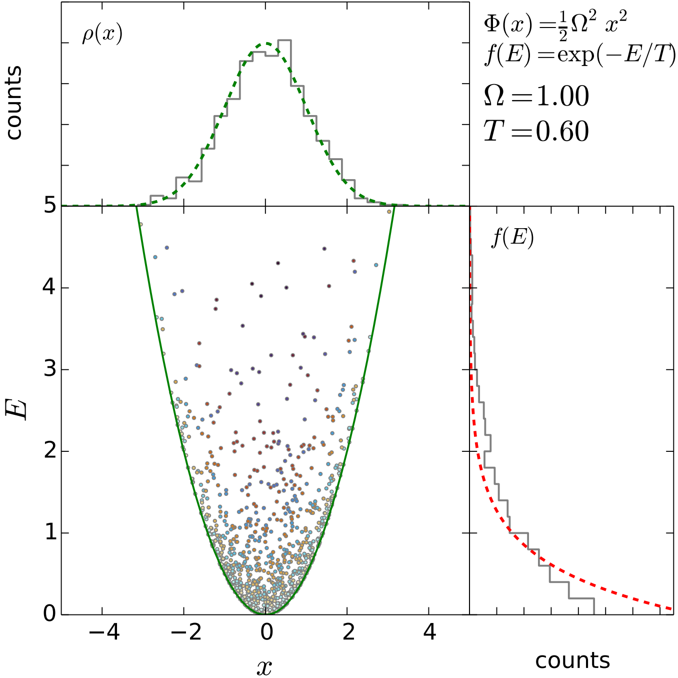
Dynamical modelling with discrete tracers



Example: particles moving in a 1d simple harmonic oscillator potential with a Maxwell–Boltzmann distribution function.

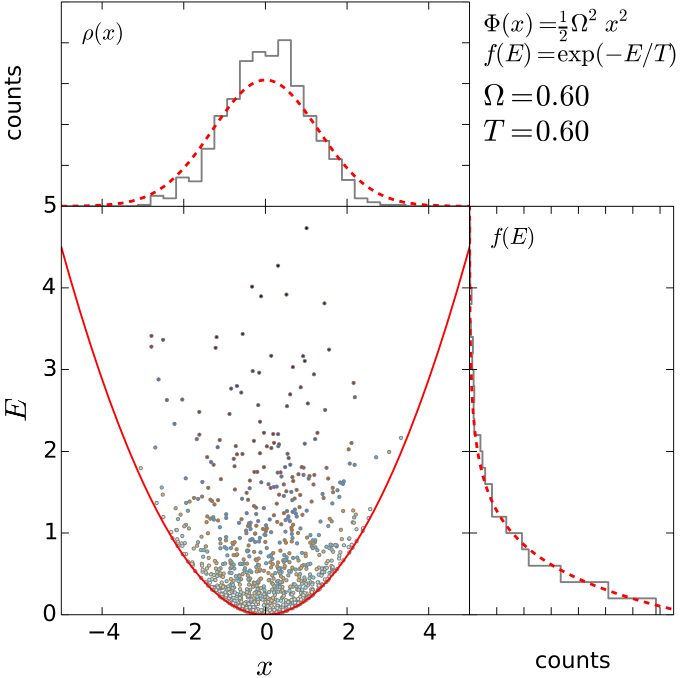
We have measured positions and velocities for $N \gg 1$ particles and want to infer the parameters of the potential (Ω) and the DF (T) that best describe the observed sample.

Dynamical modelling with discrete tracers



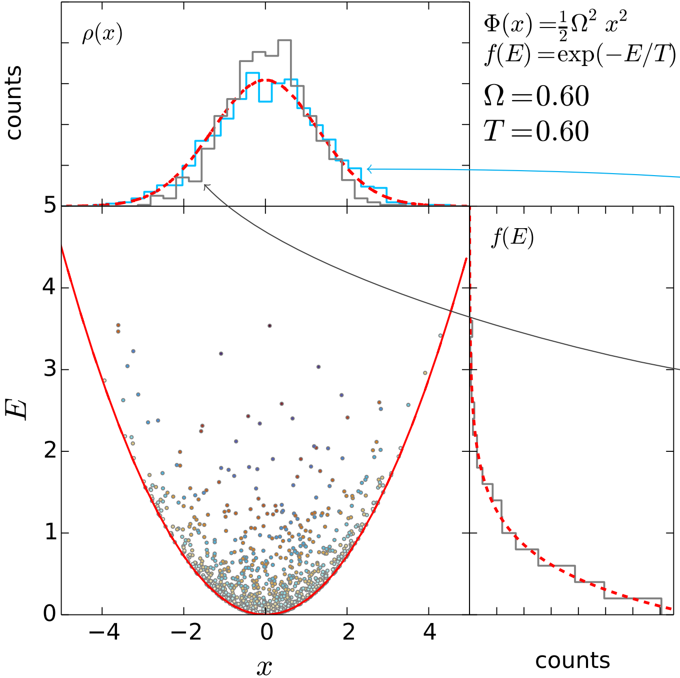
If we assume a wrong temperature T in the true potential, obviously the predicted $f(E)$ will differ from the actual distribution.

Dynamical modelling with discrete tracers



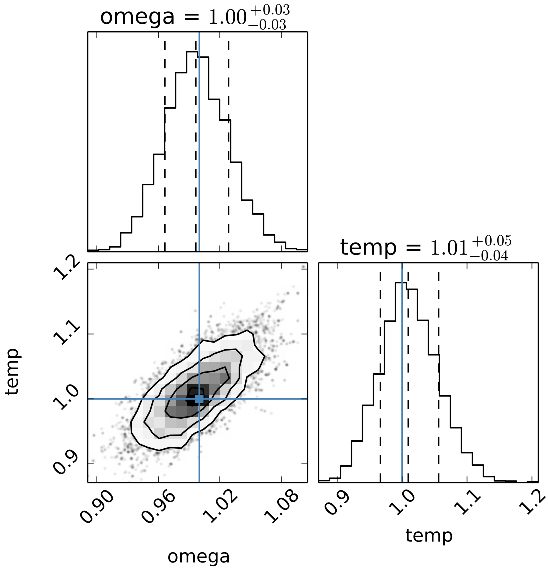
But what if we assume wrong values for both Ω and T? $f(E)$ now agrees with the observed (but incorrectly computed) energy distribution of particles, but their predicted spatial distribution should be wider: there are too many particles near $x = 0$ and too few near turnaround points ($v = 0$).

Dynamical modelling with discrete tracers



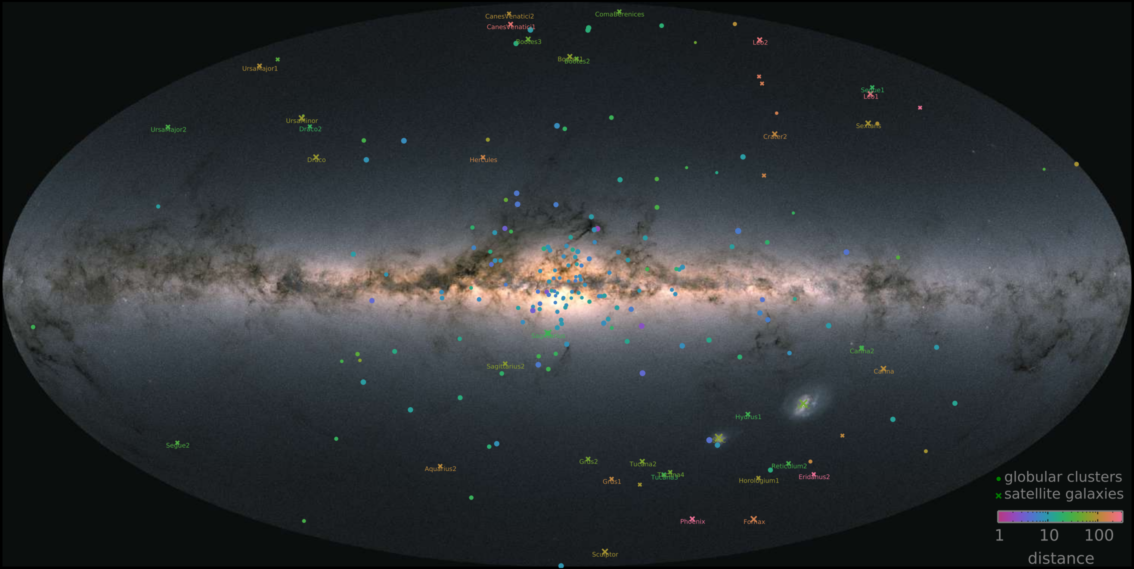
The phase-mixed population of particles predicted by the model with wrong parameters will differ from the observed distribution.

Dynamical modelling with discrete tracers



Thus we should be able to infer *both* the potential and the DF from the observed distribution of points in phase space *under the assumption of equilibrium (phase-mixedness)*.

Dynamical tracers in the Galactic halo



Dynamical tracers in the Galactic halo

~160 globular clusters:

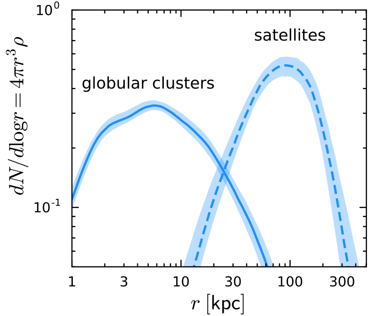
DR2 PM: Vasiliev 2019; Baumgardt+ 2019;

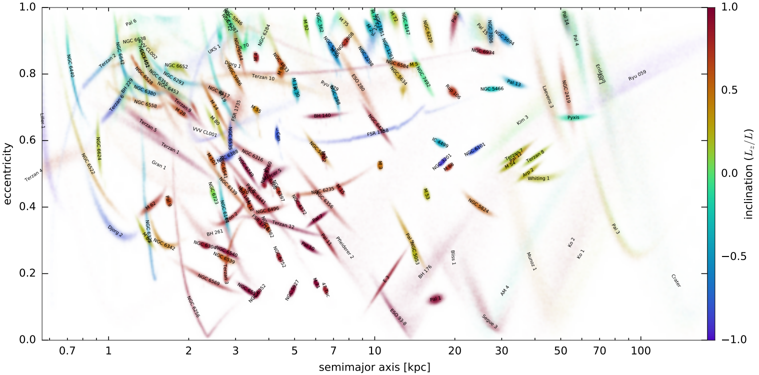
EDR3 PM: Vasiliev&Baumgardt 2021, distances: Baumgardt&Vasiliev 2021, other params: Baumgardt&Hilker 2018

~50 satellite galaxies:

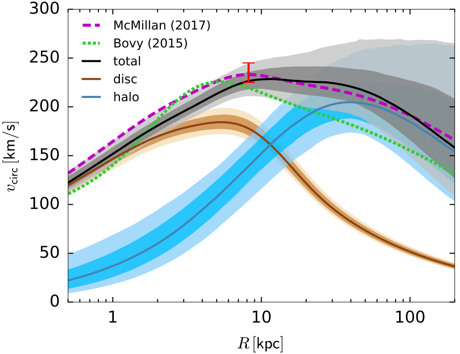
EDR3 PM: Li+ 2021; Battaglia+ 2022; Pace+ 2022;

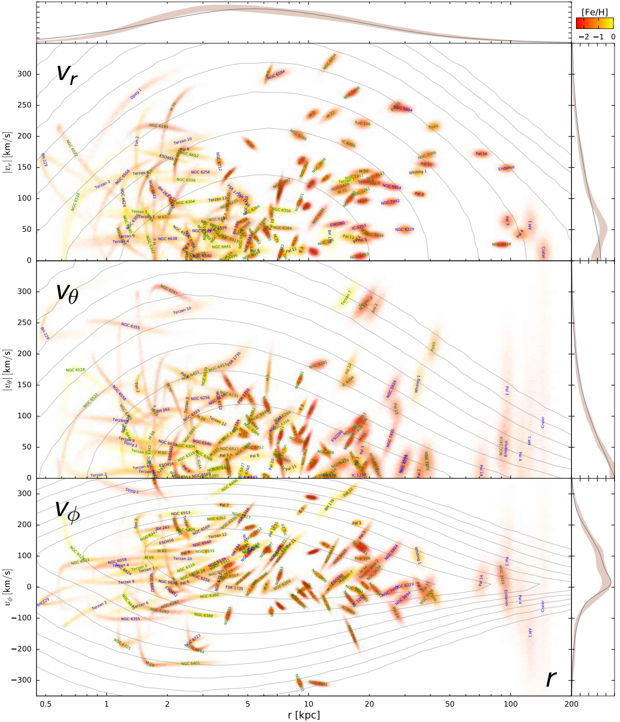
see also the Local Volume Database [Pace 2024]



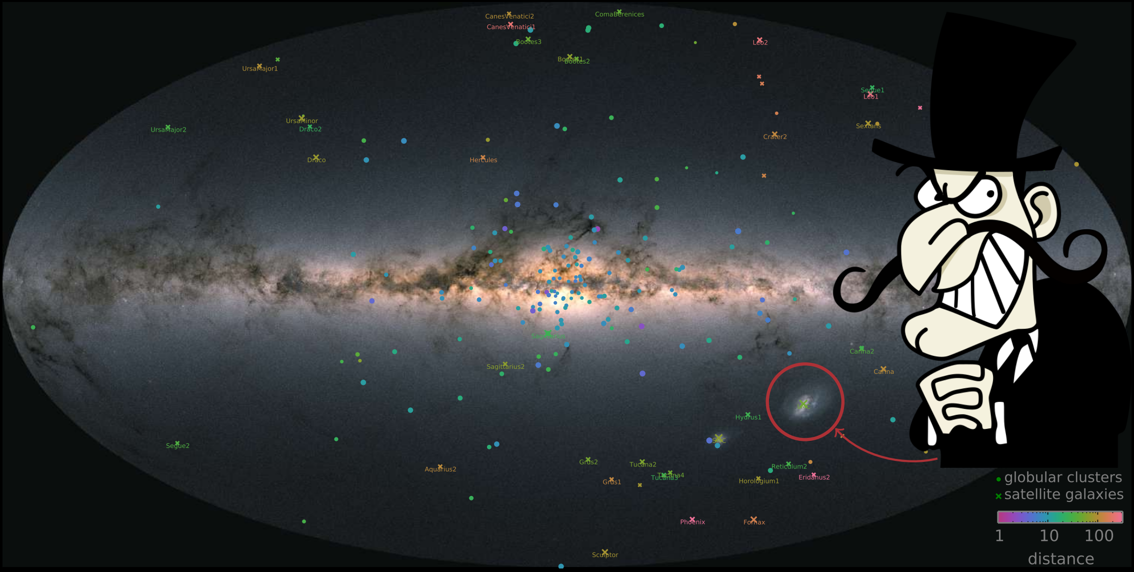


Results (DR2 version; Vasiliev 2019)





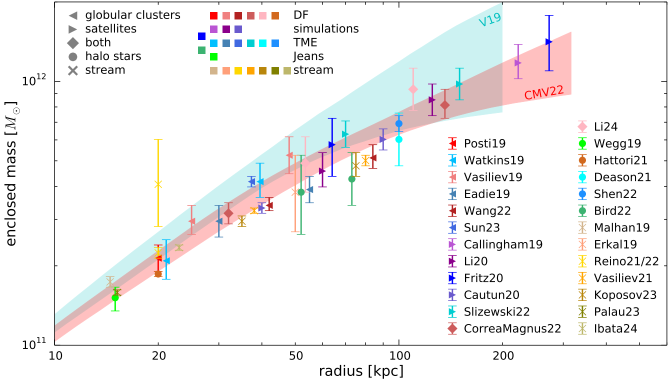
But this ignored the perturbations induced by the LMC!

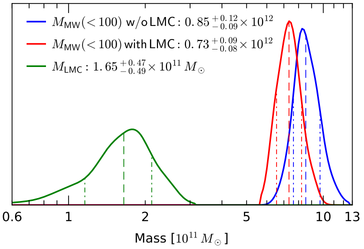


Results (EDR3 version; Correa Magnus & Vasiliev 2022)

Add an extra stage to reconstruct the original unperturbed state for any choice of Galactic potential and LMC mass:

1. Compute the past trajectories of both MW and LMC;
2. Rewind the orbits of tracers (GC and dSph) in the evolving MW+LMC potential back in time until the LMC is far enough to be ignored (at time $\hat{t} \sim$ 2–3 Gyr ago);
3. Proceed as usual, maximise $\mathcal{L} = \prod_i f(\hat{\mathbf{x}}_i, \hat{\mathbf{v}}_i)$ using pos/vel of tracers at time $\hat{t}$.

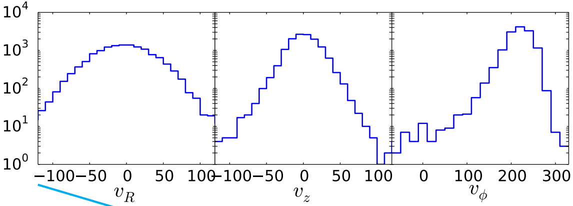


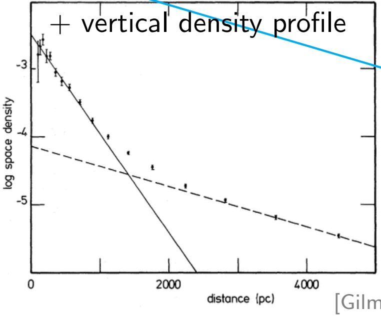


Chapter 2: Self-consistent chemodynamical Milky Way model

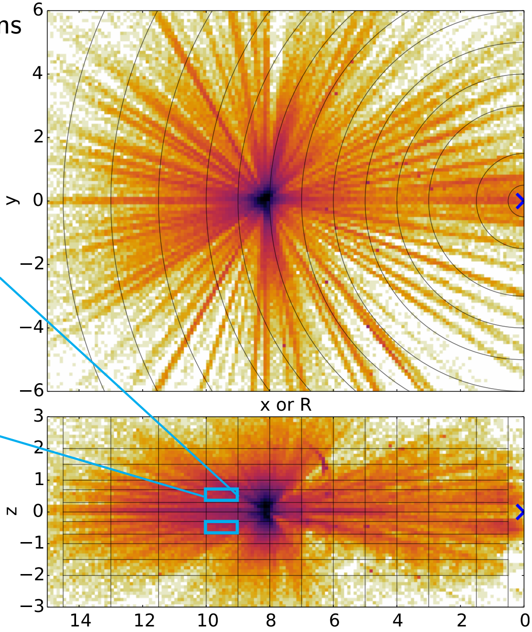
Binney & Vasiliev 2023: Gaia DR2 RVS; B&V 2024: Gaia DR3 + APOGEE DR17

Input data: histograms of velocity distributions in a few dozen spatial bins across R, z plane

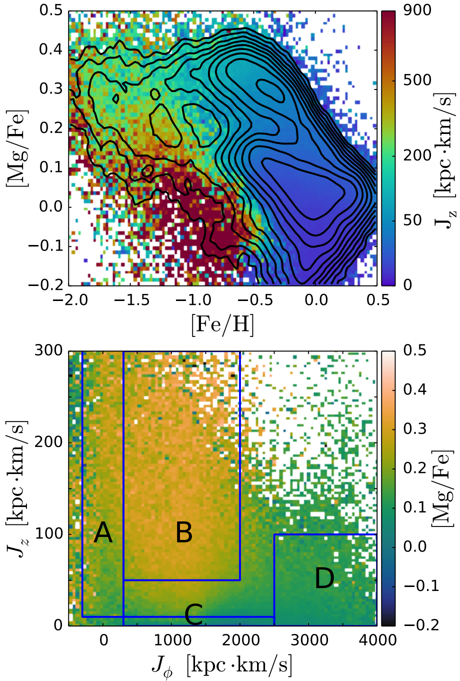




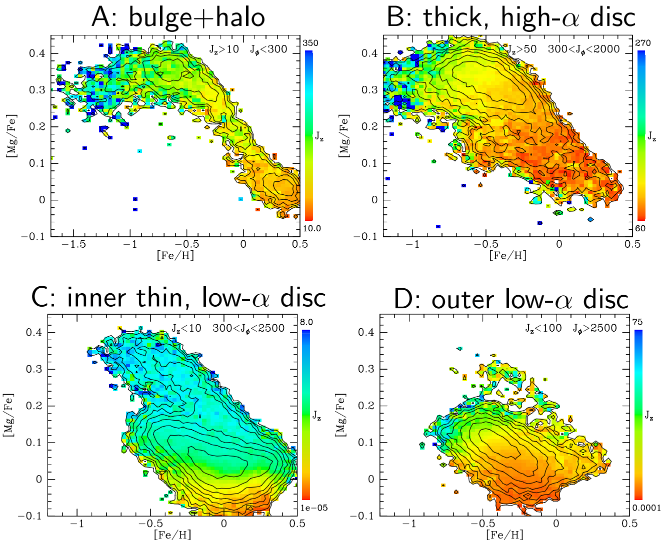
[Gilmore & Reid 1983]



Chemo-kinematic components in the Milky Way

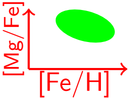


split by angular momentum J_ϕ
and vertical action J_z



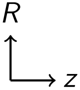
Model fitting

- Choose suitable DF families $f_k(\mathbf{J};\ \boldsymbol{\xi})$ for all galactic components (several discs, bulge, stellar and dark halo) with 6–10 free parameters $\boldsymbol{\xi}$ per component k.
- Assign a chemical DF $P_k(\mathbf{c} \mid \mathbf{J};\ \boldsymbol{\eta})$ for each stellar component ($\mathbf{c} \equiv$ [Fe/H] and [Mg/Fe], $\boldsymbol{\eta}$ are ~ 10 chemical parameters).

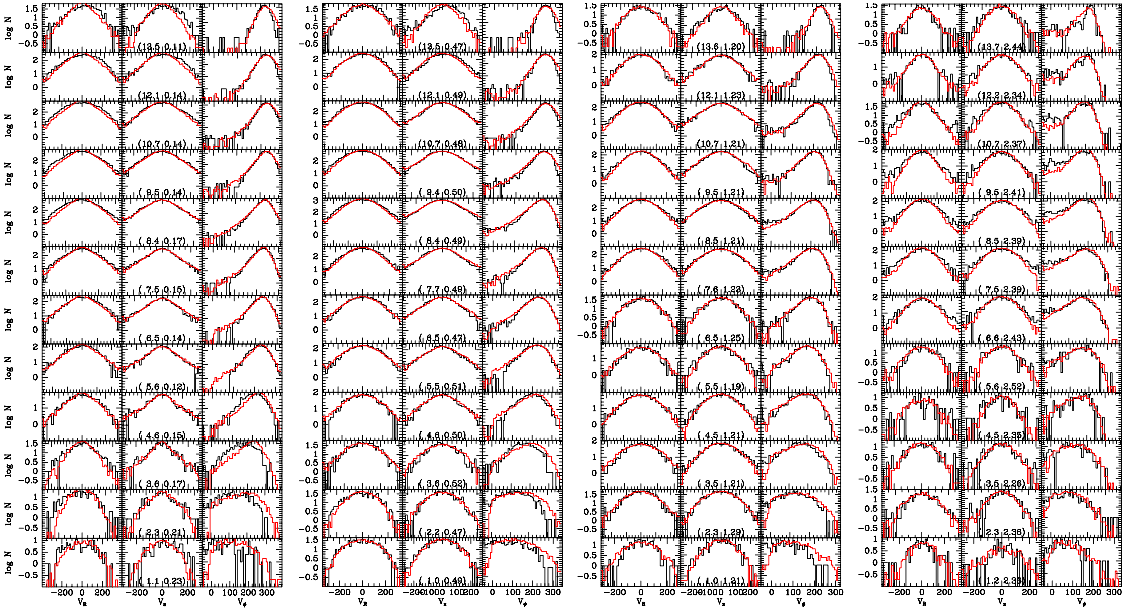


- For each choice of parameters $\boldsymbol{\xi}, \boldsymbol{\eta}$:
  - Construct a self-consistent dynamical model using the iterative method;
  - Compute velocity distributions $f(v_R),\ f(v_z),\ f(v_\phi)$ in a few dozen spatial bins;
  - Compute chemical distributions in a few dozen bins in action space;
  - Compare with observed histograms, *ignoring (freely adjusting) the overall normalization in each bin*, compute the [quasi-]likelihood $\mathcal{L}$.
- Adjust parameters and repeat (try to find the maximum-likelihood solution)...

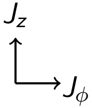
Results: kinematic distributions



fits to velocity histograms across the entire disc: not perfect, but reasonable

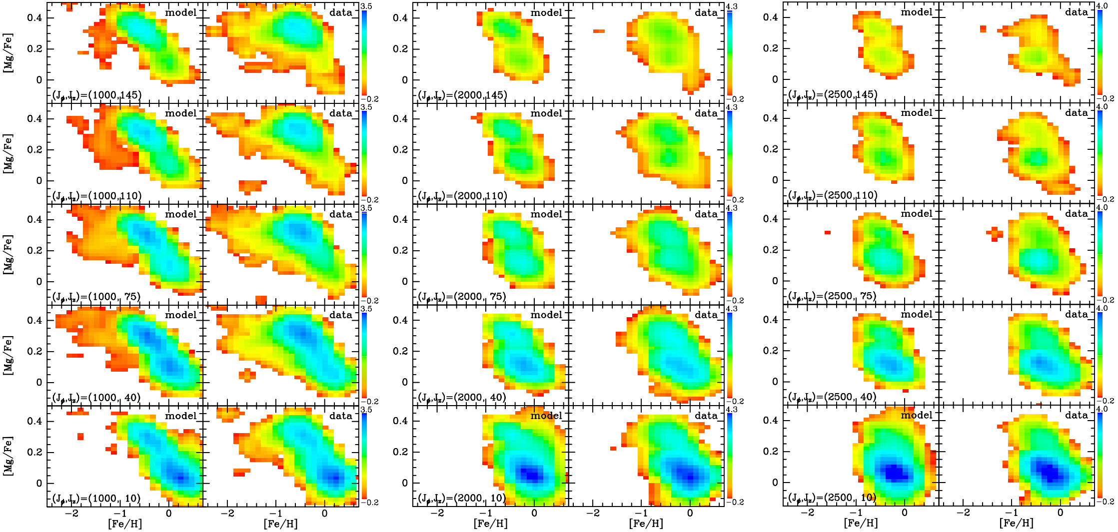


Results: chemical distributions



fits to chemical histograms [Fe/H] vs. [Mg/Fe] in 30 bins
in the J_ϕ–J_z space: qualitatively reproduce the main features
(e.g., α-poor becoming geometrically thick outside $R_\odot$)

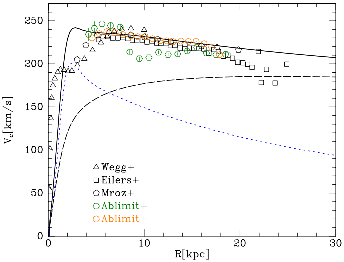
model data



Results: intrinsic properties of the model

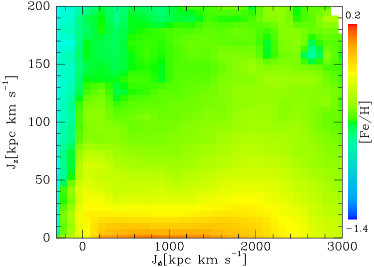
Structural properties are well fitted, chemical fits still need improvement

circular-velocity curve

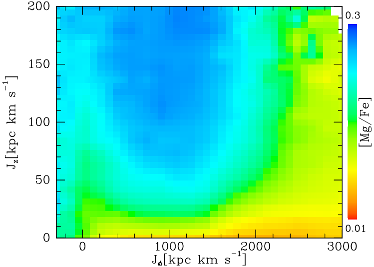


[Fe/H]

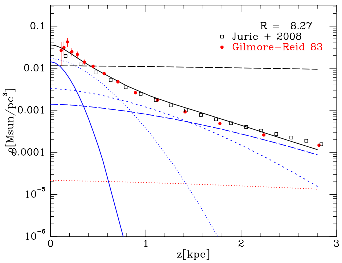
data



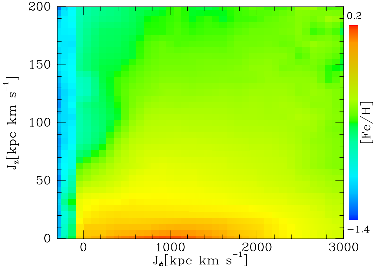
[Mg/Fe]

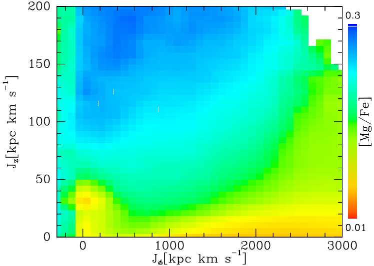


vertical density profile

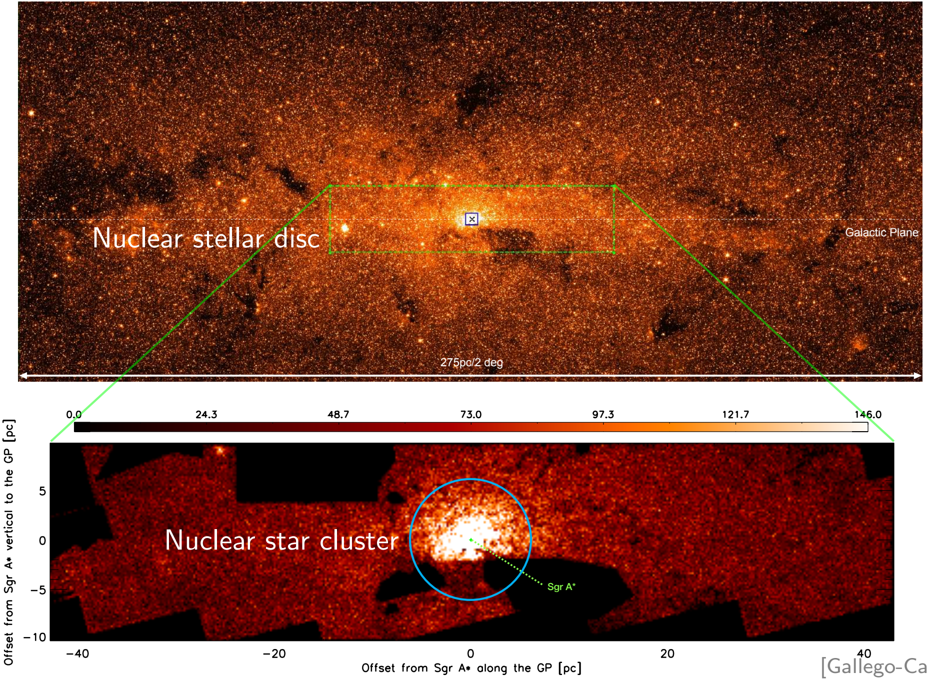


model



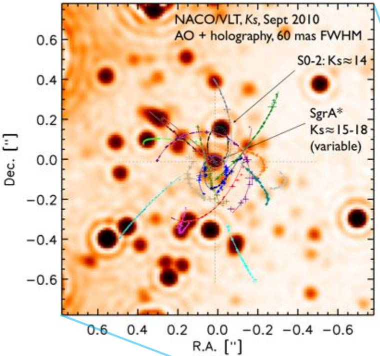


Chapter 3: Self-consistent model of the nuclear star cluster

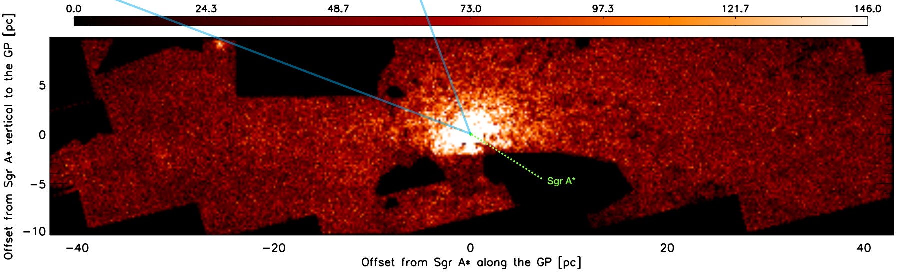


[Gallego-Cano+ 2020]

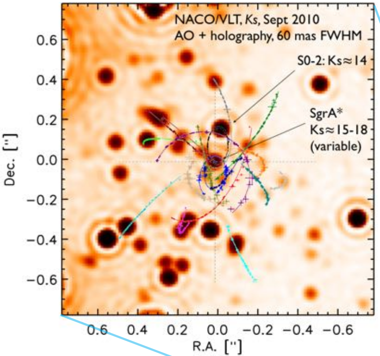
Chapter 3: Self-consistent model of the nuclear star cluster



orbits of
$\mathcal{O}(10)$ S-stars
[Genzel+; Ghez+;
GRAVITY collab.]



Chapter 3: Self-consistent model of the nuclear star cluster



orbits of

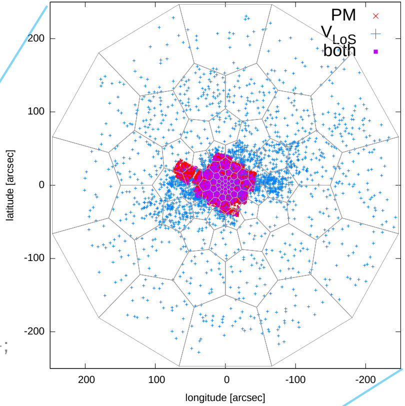
$\mathcal{O}(10)$ S-stars

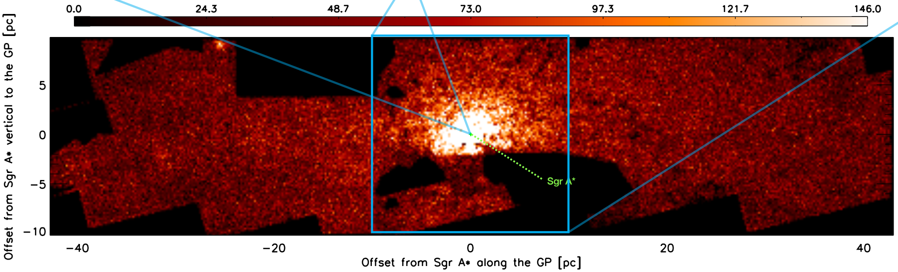
[Genzel+; Ghez+; GRAVITY collab.]

single-epoch kinematic snapshot

few$\times 10^3$ stars

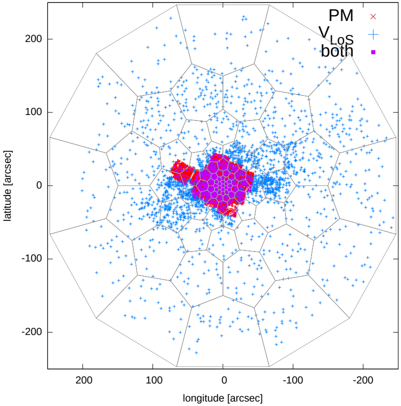
[Schödel+; Fritz+; Feldmeier+]





Model fitting

- Adopt parameters **p** describing the NSC DF (e.g., mass, inner/outer slope, anisotropy, rotation) and BH mass $M_\bullet$; the NSD DF is kept fixed.
- Construct the self-consistent model .
- Compute the 1d velocity distribution functions (VDFs) $\mathfrak{f}^{(b)}(v_{x,y,z}\,|\,\mathbf{x}^{(b)})$ in ~ 100 spatial bins.
- Evaluate the likelihood $\mathcal{L}$ of the observed dataset given the model VDFs (sum of NSC and NSD).
- Repeat with a different choice of parameters **p** in the MCMC loop.



Note that there is no binning of observational data! (unlike in Chapter 2).
bins are only used to construct spatially varying model VDFs,
and each star's contribution to the likelihood is summed up separately.

We also do not input any information about the density profile (although we could..)

Digression: inferring the potential from kinematic data

Method 1:

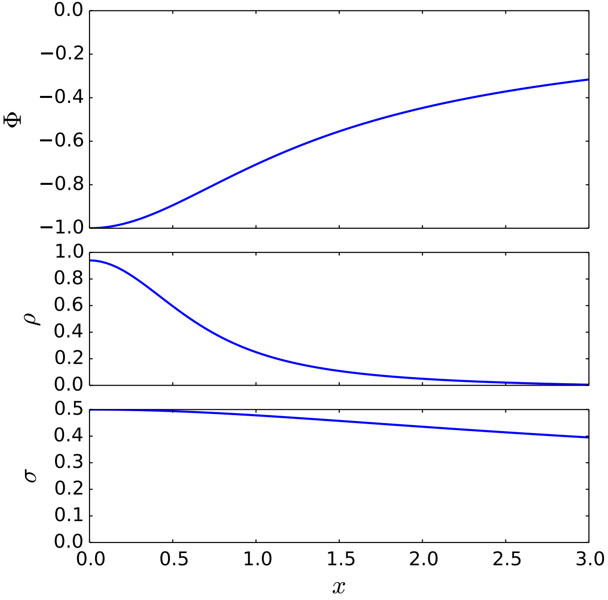
Jeans equation(s):

$\rho(x)$, $\sigma(x) \Rightarrow \Phi(x)$.

1d example:

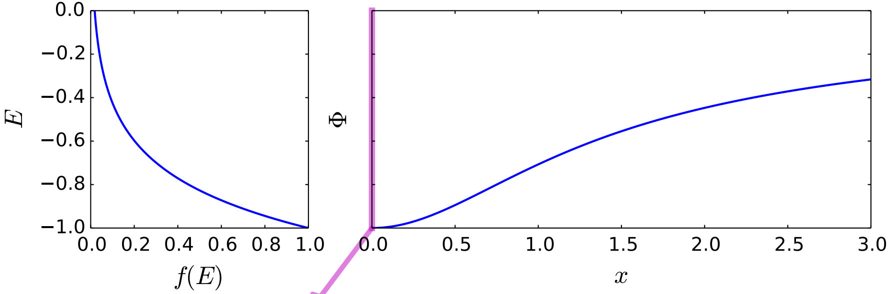
$$\underbrace{\frac{\mathrm{d}\left(\rho\,\sigma^2\right)}{\mathrm{d}x}}_{\text{pressure gradient}} + \rho\,\underbrace{\frac{\mathrm{d}\Phi}{\mathrm{d}x}}_{\text{gravitational force}} = 0$$

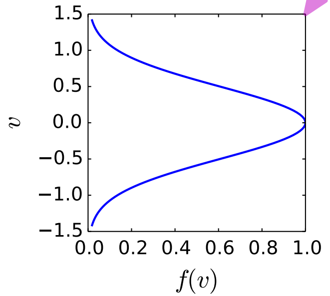
(hydrostatic equilibrium).



Digression: inferring the potential from kinematic data

Method 2: DF





$$f(x, v) \Leftrightarrow f\left(E = \Phi(x) + \tfrac{1}{2}v^2\right)$$

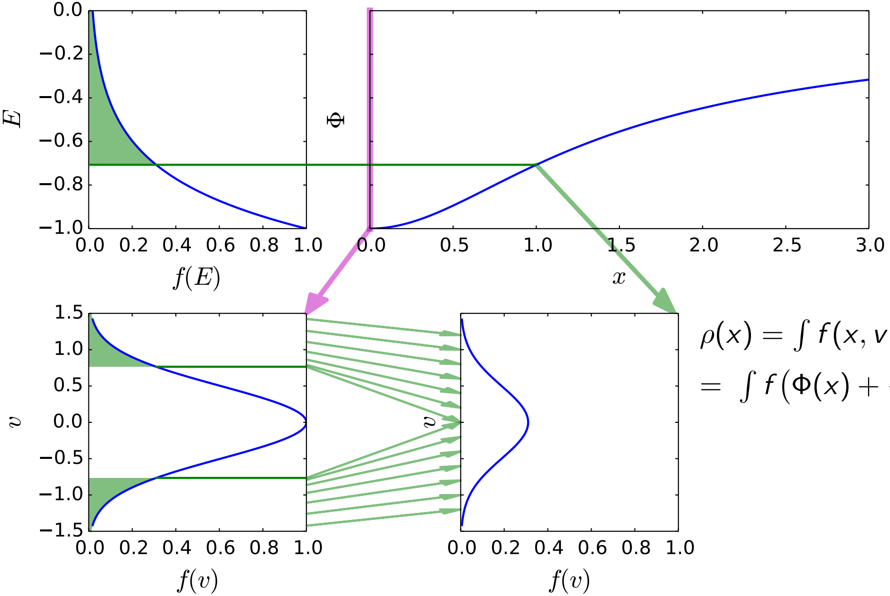
for example,

$$f(E) = f_0 \exp\left(-\frac{E}{\sigma^2}\right),$$

$$f(x, v) = f_0 \exp\left(-\frac{\Phi(x)}{\sigma^2}\right) \exp\left(-\frac{1}{2}\frac{v^2}{\sigma^2}\right)$$

Digression: inferring the potential from kinematic data

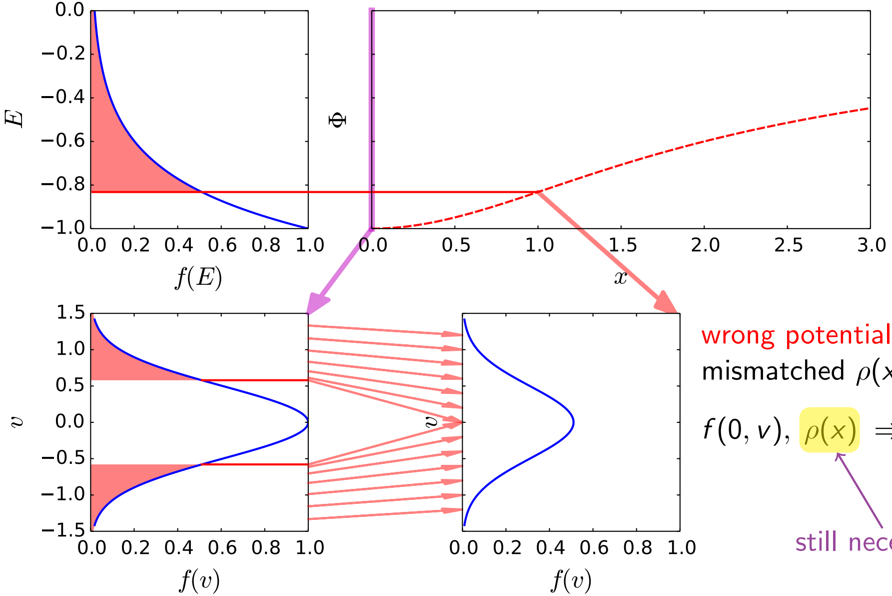
Method 2: DF



$$\rho(x) = \int f(x, v)\, \mathrm{d}v$$
$$= \int f\left(\Phi(x) + \tfrac{1}{2}v^2\right)\, \mathrm{d}v$$

Digression: inferring the potential from kinematic data

Method 2: DF



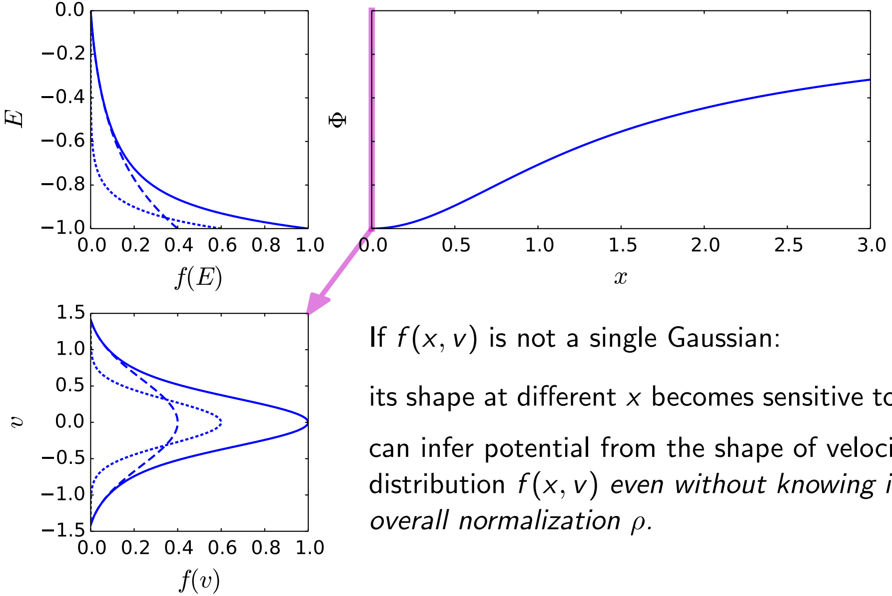
wrong potential $\Rightarrow$ mismatched $\rho(x)$.

$f(0, v)$, $\rho(x)$ $\Rightarrow$ $\Phi(x)$.

still necessary!

Digression: inferring the potential from kinematic data

Method 2: DF



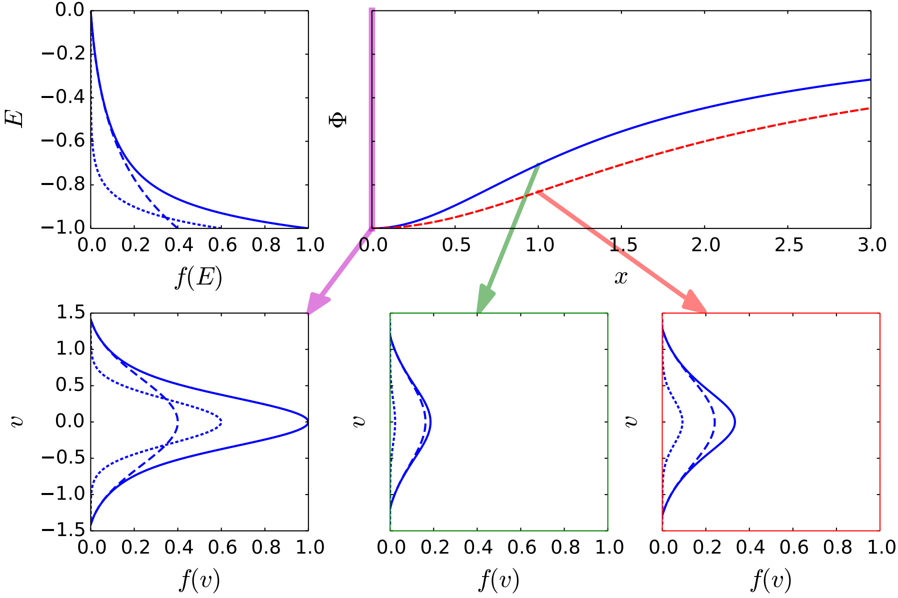
If $f(x, v)$ is not a single Gaussian:

its shape at different x becomes sensitive to Φ $\Rightarrow$

can infer potential from the shape of velocity distribution $f(x, v)$ *even without knowing its overall normalization* ρ*.*

Digression: inferring the potential from kinematic data

Method 2: DF

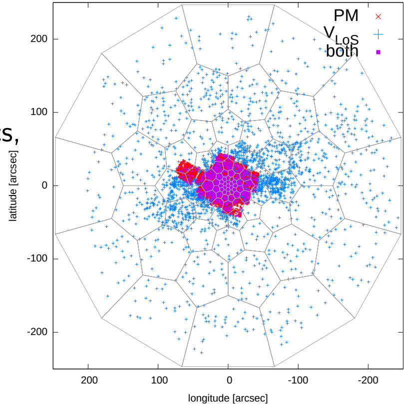


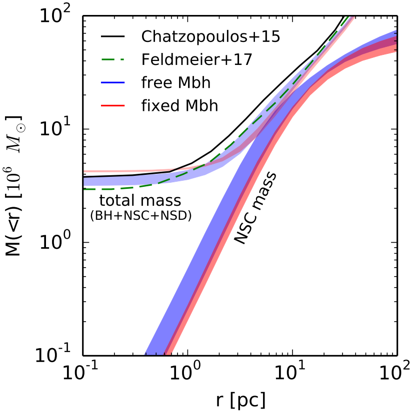
Results

Black hole mass $M_\bullet = (3.5 \pm 0.3) \times 10^6\, M_\odot$
("true" value from orbits of S-stars: $4.3 \times 10^6\, M_\odot$).

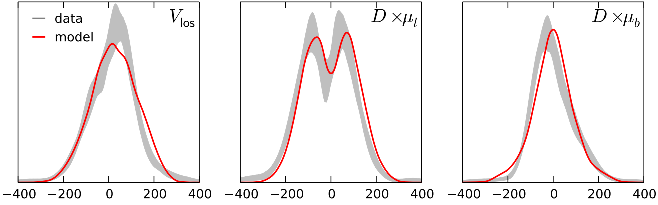
Stellar density profile is inferred purely from kinematics, and is not too unrealistic.

Fits using only v_los data are unable to constrain $M_\bullet$, despite these data covering a larger region.

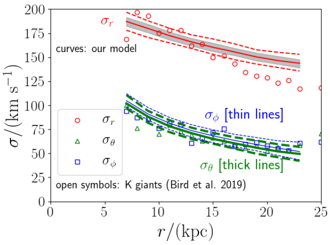




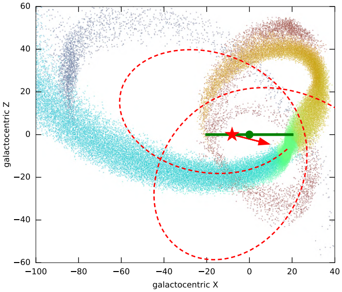
VDFs in one of many spatial bins



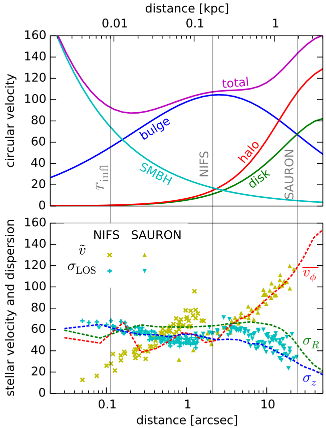
Other applications



DF models of RR Lyrae in the Milky Way halo [Hattori+ 2021]



modelling the Sagittarius stream [Vasiliev+ 2021]



measuring SMBH masses in external galaxies using the Schwarzschild orbit-superposition code FORSTAND [Merrell+ 2023]

...and many more!

AGAMA is a powerful tool for various tasks in galactic dynamics – perhaps you may find it useful in your projects?