Schwarzschild modelling of barred galaxies and supermassive black holes

Eugene Vasiliev

Institute of Astronomy, Cambridge

Dynamical Reconstruction of Galaxies workshop

Lorentz Center, 19 February 2020

Synopsis

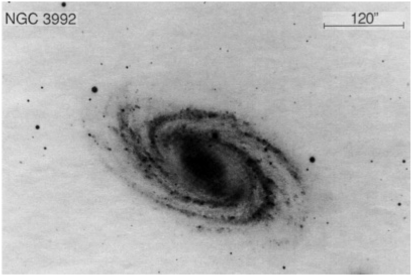
- Motivation
  - Barred galaxies are ubiquitous ($\sim 50\%$ of disk galaxies)
  - Complex morphology and kinematics
  - Interplay between bars and supermassive black holes
- Modelling approaches for barred galaxies
- Schwarzschild's method in brief
- Results for barred galaxies and supermassive black holes
- Open questions
  - Deprojection uncertainties
  - Intrinsic degeneracies in potential determination
  - Statistical challenges in non-parametric modelling

Modelling approaches for barred galaxies: response models

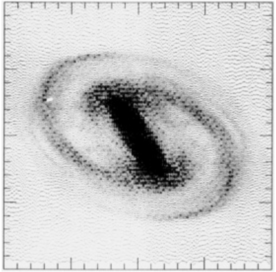
[Contopoulos & Grosbøl 1986, 1988; Patsis+ 1991; Kaufmann & Contopoulos 1996]

2d response models:

- assume parameters for potential, pattern speed, etc.
- construct the network of periodic orbits
- populate nearby orbits and compute their surface density
- compare morphological features with observations
- vary the parameters until a good match is found



observed galaxy

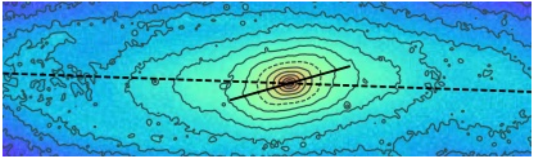


response model

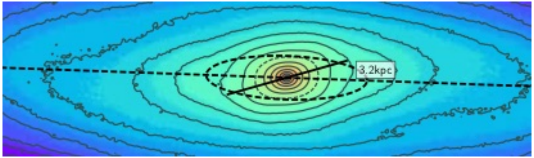
Modelling approaches for barred galaxies: made-to-measure

Introduced by Syer & Tremaine 1996,
grown up and flourished in Ortwin Gerhard's group [Bissantz+ 2004, de Lorentzi+ 2007, Portail+ 2015; Blaña+ 2019];
several other implementations exist [Dehnen 2009; Long & Mao 2012; Hunt & Kawata 2013; Malvido & Sellwood 2015]

Idea: evolve an N-body model while adjusting particle weights to match the observables (density and kinematics)



observed galaxy (M31)



M2M model [Blaña+ 2019]

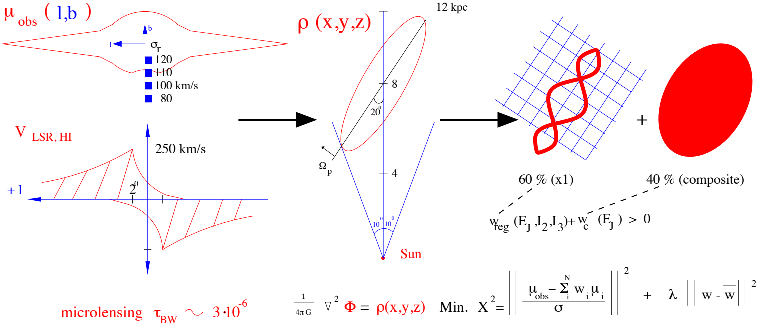
Modelling approaches for barred galaxies: orbit superposition

Introduced by Martin Schwarzschild (1979) in a theoretical context;

theoretical study of kinematics of 2d barred galaxies [Pfenniger 1984, 1985];

application to MW bar [Zhao 1996; Häfner+ 2000; Wang+ 2012]:

density taken from deprojected COBE star counts; kinematics fitted to a collection of observations (BRAVA survey, $v_{\rm los}$ and PM in Baade's window, etc.); other constraints: microlensing depth, gas terminal velocities.

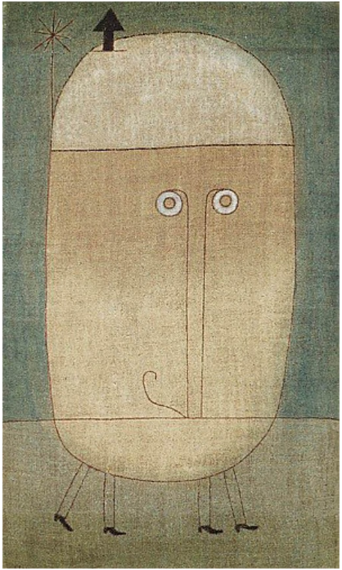


$$\frac{1}{4\pi G}\nabla^2\Phi = \rho(x,y,z) \quad \mathrm{Min.}\ X^2 = \left|\left| \frac{\mu_{obs} - \sum_i^N w_i \mu_i}{\sigma} \right|\right|^2 + \lambda \left|\left| w - \overline{w} \right|\right|^2$$

[Zhao 1996]

Modelling approaches for barred galaxies: orbit superposition

Schwarzschild



Pfenniger

Zhao

Schwarzschild's orbit-superposition method: basics

Introduced by Schwarzschild (1979) as a practical approach
for constructing self-consistent triaxial models with prescribed $\rho(\mathbf{x}) \Leftrightarrow \Phi(\mathbf{x})$.

integrals of motion

To invert the equation $\rho(\mathbf{x}) = \iiint f\left(\mathcal{I}\left[\mathbf{x}, \mathbf{v} \mid \Phi\right]\right)\, d^3\mathbf{v}$,

discretize both the density profile and the distribution function:

$\rho(\mathbf{x}) \implies$ cells of a spatial grid; mass of each cell is $M_c = \iiint\limits_{\mathbf{x}\in V_c} \rho(\mathbf{x})\, d^3x$;

$f(\mathcal{I}) \implies$ collection of orbits with unknown weights [to be determined]:

$$f(\mathcal{I}) = \sum_{k=1}^{N_{\rm orb}} w_k\, \delta(\mathcal{I} - \mathcal{I}_k)$$

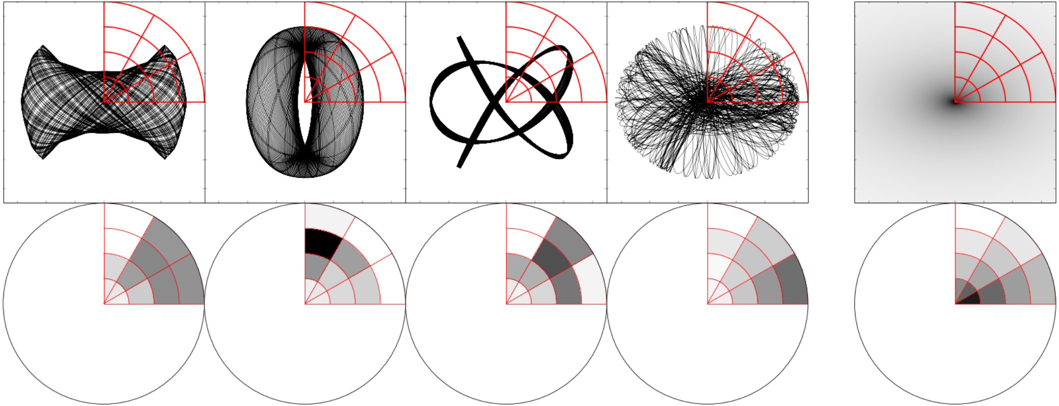
each orbit is a delta-function in the space of integrals of motion

adjustable weight of each orbit

Schwarzschild's orbit-superposition method: self-consistency

orbits in the model

target density



discretized orbit density

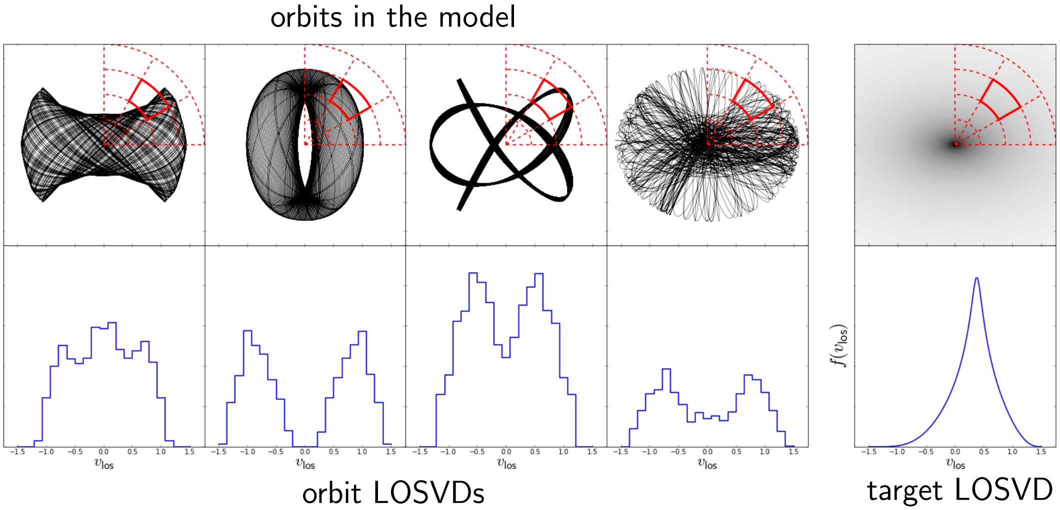
(fraction of time t_{kc} that k-th orbit spends in c-th cell)

discretized density

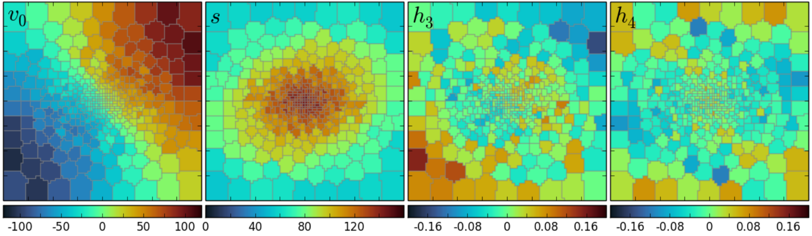
(mass M_c in grid cells)

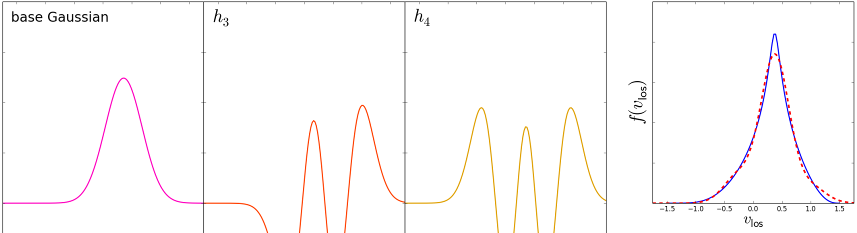
For each c-th cell we require $\sum_k w_k\, t_{kc} = M_c$, where $w_k \geq 0$ is orbit weight

Schwarzschild's orbit-superposition method: kinematics



Schwarzschild's orbit-superposition method: kinematics





Gauss–Hermite parametrization of LOSVDs [van der Marel & Franx 1993; Gerhard 1993]

Schwarzschild's orbit-superposition method: fitting procedure

- Assume some potential $\Phi(\mathbf{x})$
  (e.g., from the deprojected luminosity profile plus parametric DM halo or SMBH)
- Construct the orbit library in this potential:
  for each k-th orbit, store its contribution to the discretized density profile
  t_{kc}, $c = 1..N_\mathrm{cell}$ and to the kinematic observables u_{kn}, $n = 1..N_\mathrm{obs}$
- Solve the constrained optimization problem to find orbit weights w_k:

$$\text{minimize } \chi^2 + \mathcal{S} \equiv \sum_{n=1}^{N_\mathrm{obs}} \left(\frac{\sum_{k=1}^{N_\mathrm{orb}} w_k\, u_{kn} - U_n}{\delta U_n} \right)^2 + \mathcal{S}(\{w_k\})$$

regularization term

observational constraints

their uncertainties

$$\text{subject to } w_k \ge 0, \quad k = 1..N_\mathrm{orb},$$

$$\sum_{k=1}^{N_\mathrm{orb}} w_k\, t_{kc} = M_c, \quad c = 1..N_\mathrm{cell}$$

density constraints (cell masses)

- Repeat for different choices of potential and find the one that has lowest χ^2

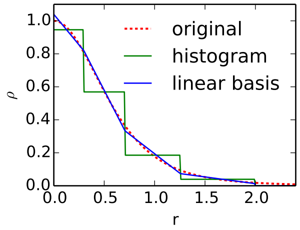
Schwarzschild's orbit-superposition method: implementations

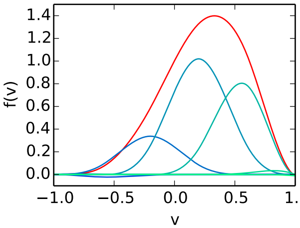
Several commonly used independent implementations of the method:

- theoretical studies in triaxial geometry: Schwarzschild 1979, 1993; Pfenniger 1984; Statler 1987; Merritt & Fridman 1996; Siopis & Kandrup 2000; Vasiliev 2013
- spherical codes: Richstone & Tremaine 1984; Rix+ 1997; Jalali & Tremaine 2010; Breddels & Helmi 2013; Kowalczyk+ 2017
- axisymmetric: "Leiden" [van der Marel, Cretton, Cappellari, ...– since 1998]
- axisymmetric: "Nukers" [Gebhardt, Richstone, Kormendy, ...– since 2000]
- axisymmetric: "MasMod" [Valluri, Merritt, Emsellem – since 2004]
- triaxial/Milky Way bar: Zhao, Wang, Mao 1996, 2012
- triaxial: van den Bosch, van de Ven, de Zeeuw, Zhu, ...– since 2008 $\Rightarrow$ "Dynamite"
- triaxial: "Forstand" [Vasiliev & Valluri 2020]

New implementation of Schwarzschild's method: highlights

- arbitrary geometry (from spherical to triaxial), arbitrary density profiles ($\Rightarrow$ flexible Poisson solver)
- rotating frame ($\Rightarrow$ triaxial bars)
- random sampling of initial conditions for orbits
- several choices for 3d intrinsic density constraints (incl. piecewise-linear shape elements)
- representation of the 3d observational datacube $(X, Y, v_{\rm los})$ in terms of B-splines
- either Gauss–Hermite moments or a full LOSVD fitting
- very efficient quadratic optimization solver
- Publicly available as part of AGAMA library for dynamical modelling



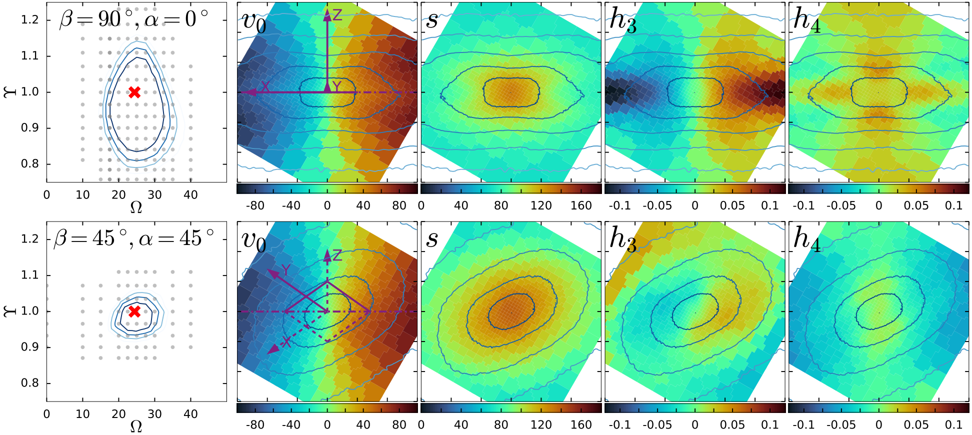


New implementation of Schwarzschild's method: highlights

- arbitrary geometry (from spherical to triaxial), arbitrary density profiles ($\Rightarrow$ flexible Poisson solver)
- rotating frame ($\Rightarrow$ triaxial bars)
- random sampling of initial conditions for orbits
- several choices for 3d intrinsic density constraints (incl. piecewise-linear shape elements)
- representation of the 3d observational datacube (X, Y, v_{los}) in terms of B-splines
- either Gauss–Hermite moments or a full LOSVD fitting
- very efficient quadratic optimization solver
- Publicly available as part of AGAMA library for dynamical modelling

Kazimir Malevich, "Suprematism"

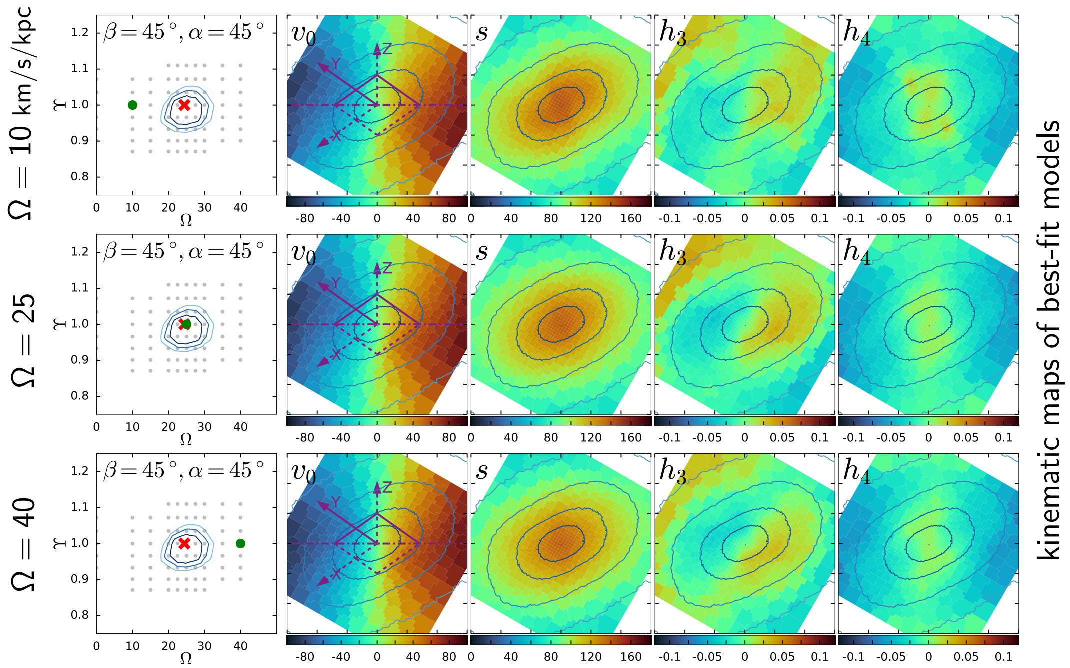
Recovery of bar pattern speed in Schwarzschild models



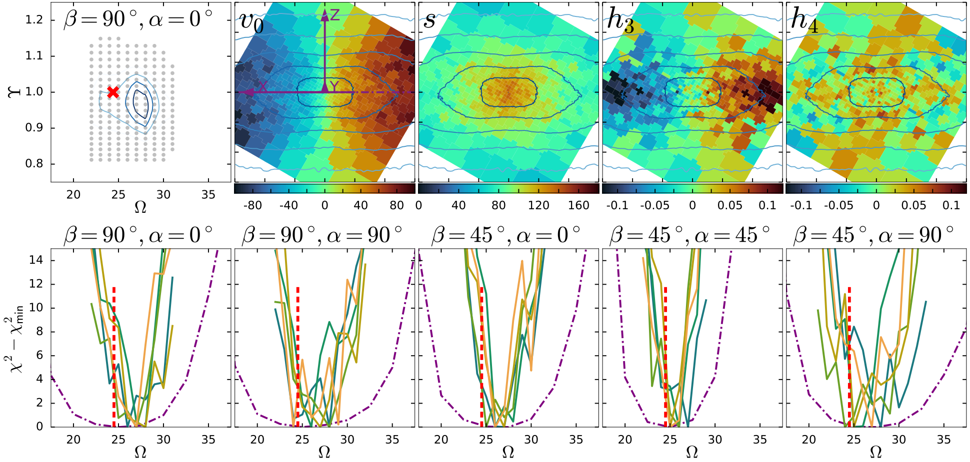
Mock data from a barred *N*-body model of the Milky Way [Fragkoudi+ 2017]

Here assumed a known 3d shape of the density profile,
varied two parameters: pattern speed Ω and mass-to-light ratio Υ

Recovery of bar pattern speed in Schwarzschild models



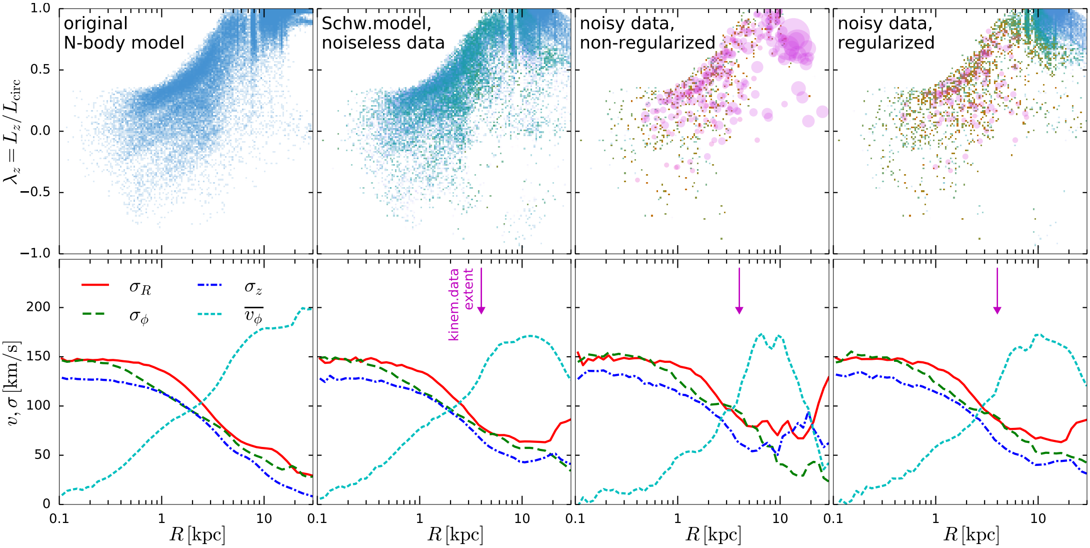
Recovery of bar pattern speed in Schwarzschild models



5 bar orientations, 5 different realizations of noise in each case;
the correct pattern speed is recovered to within 10%, even in “symmetric” cases, which are not suitable for the Tremaine–Weinberg method

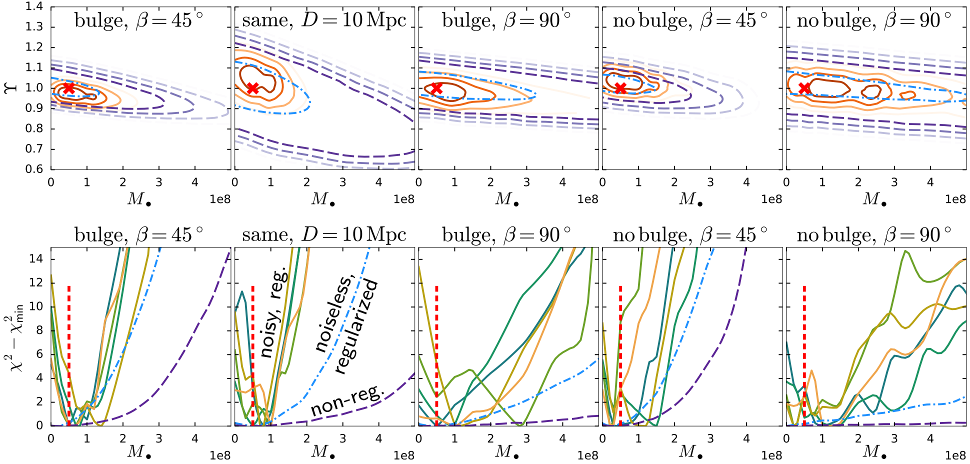
Recovery of internal kinematics in Schwarzschild models

Distribution of orbits in radius vs. "circularity" space



Radial profiles of mean velocity and its dispersion

Recovery of black hole masses in Schwarzschild models

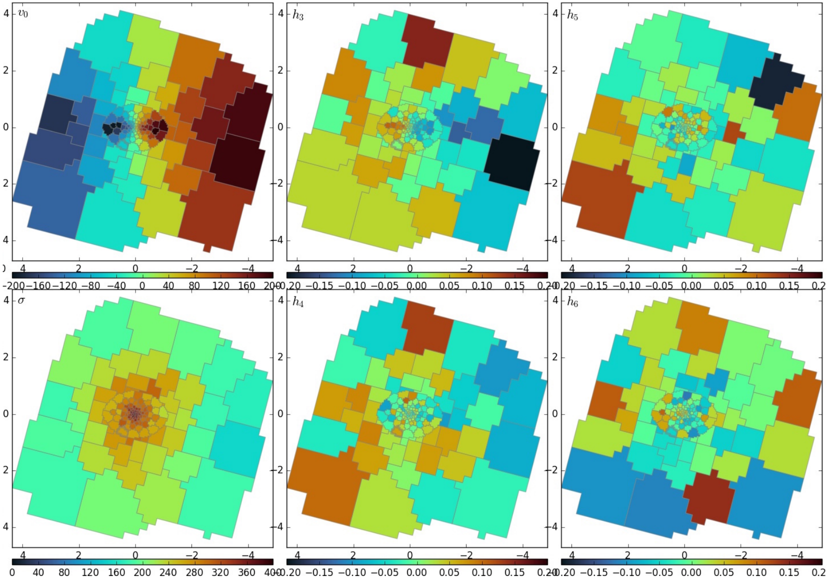


Mock data: Milky Way-size axisymmetric disk galaxy with or without a bulge, and a central black hole with mass $M_\bullet = 10^{-3}\,M_{\rm gal} = 5\times10^7\,M_\odot$, placed at 10 or 20 Mpc; two kinematic datasets – low-res (FoV $1'\times1'$, PSF width $1''$) and high-res ($2''\times2''$, $0.1''$).

Large degeneracies, noisy χ^2 contours!

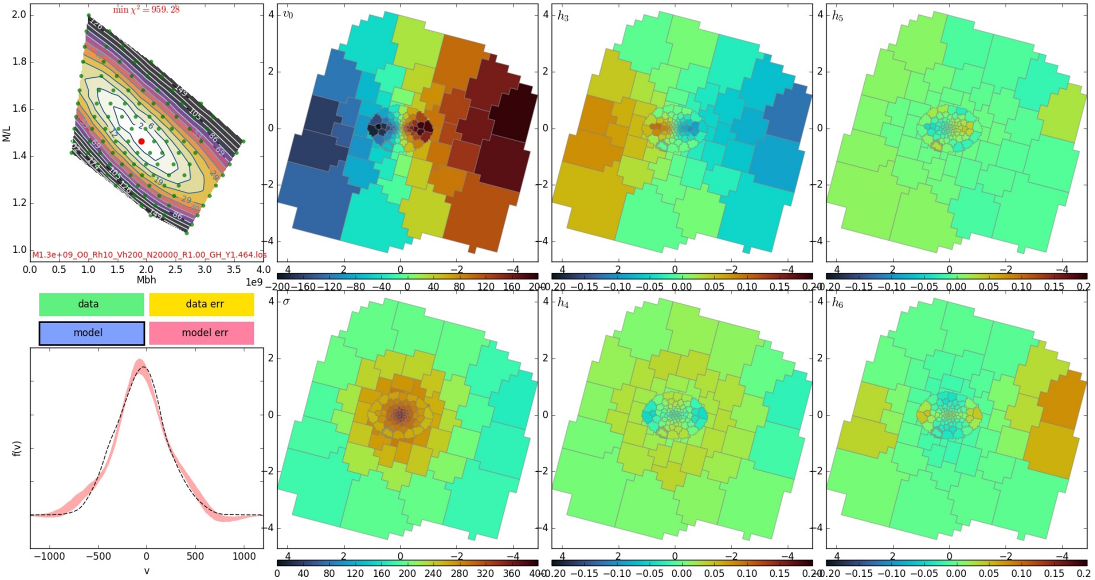
Example of Schwarzschild modelling of a real galaxy

Galaxy: PGC 12257
Distance: 70 Mpc
Comparative study led by Jonelle Walsh, using five different modelling codes



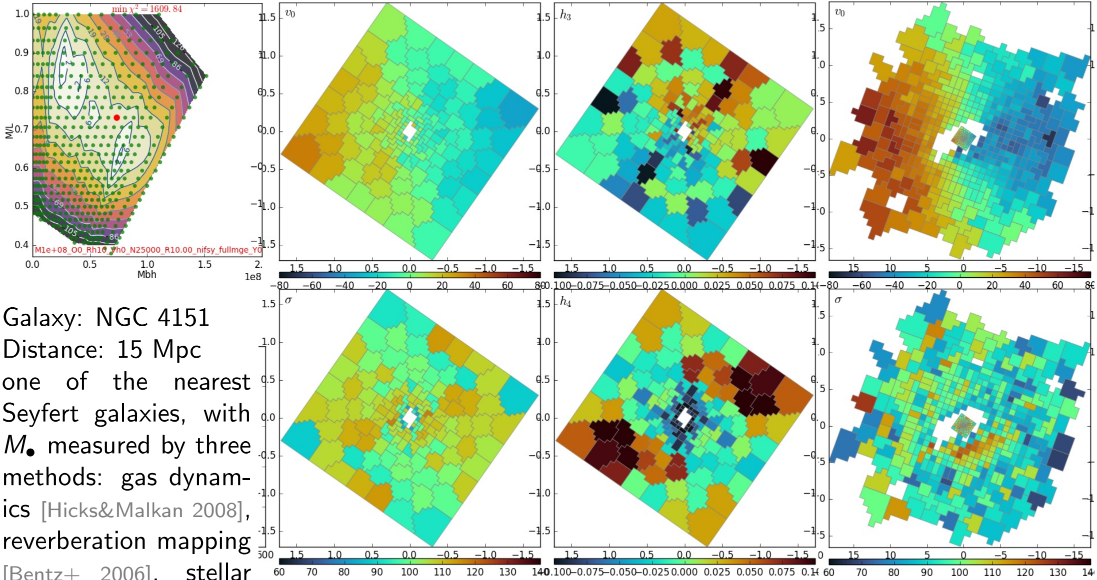
observations (two kinematic datasets: NIFS and LRS2)

Example of Schwarzschild modelling of a real galaxy



best-fit model

Example of Schwarzschild modelling of another galaxy



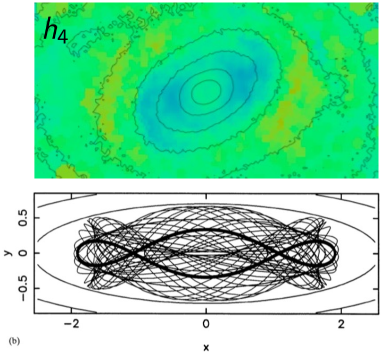
Galaxy: NGC 4151
Distance: 15 Mpc
one of the nearest Seyfert galaxies, with $M_\bullet$ measured by three methods: gas dynamics [Hicks&Malkan 2008], reverberation mapping [Bentz+ 2006], stellar dynamics [Onken+ 2014].

observations: NIFS (small-scale) and SAURON (large-scale)

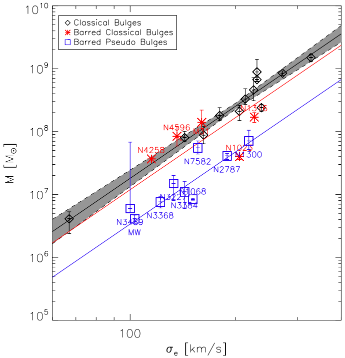
Biases in dynamical black hole masses in barred galaxies

Ignoring the presence of a bar may bias $M_\bullet$ upward:

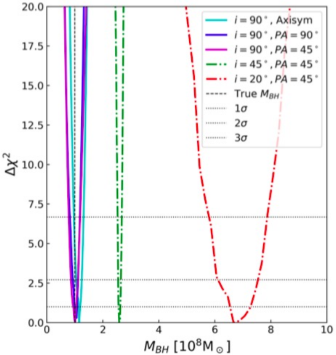
- A bar seen end-on has a higher σ mimicking a large BH [e.g., Gerhard 1988]
- Large $M_\bullet$ creates high-velocity tails (positive h_4) [van der Marel 1994], while bars have negative h_4 [e.g., Bureau&Athanassoula 2005; Debattista+ 2005]; in combination with high σ this results in overestimation of $M_\bullet$ [Brown+ 2013]



[Gerhard 1988]



[Hartmann+ 2014]



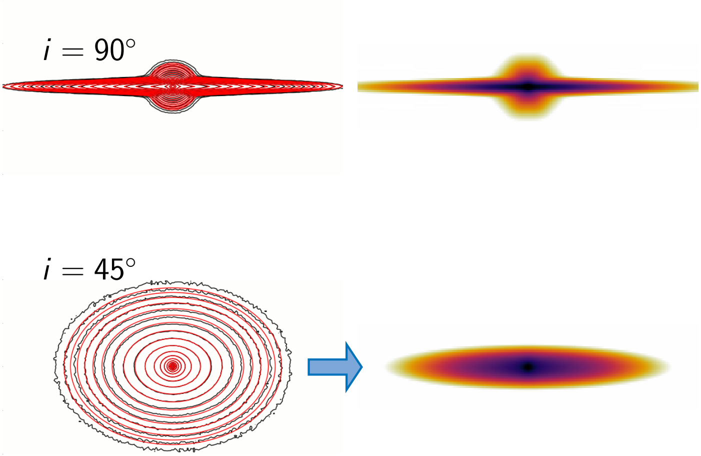
[Valluri&Vasiliev, in prep.]

Uncertainties and biases in deprojection

Deprojection is **not unique** even in the axisymmetric (except edge-on) case!

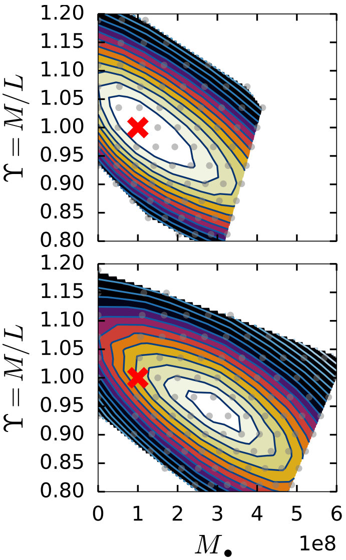
[Kochanek & Rybicki 1996; Gerhard & Binney 1996]

Multi-Gaussian expansion gives only one possible deprojection, but not necessarily a good one.



actual density profile

deprojected from MGE



Uncertainties and biases in deprojection

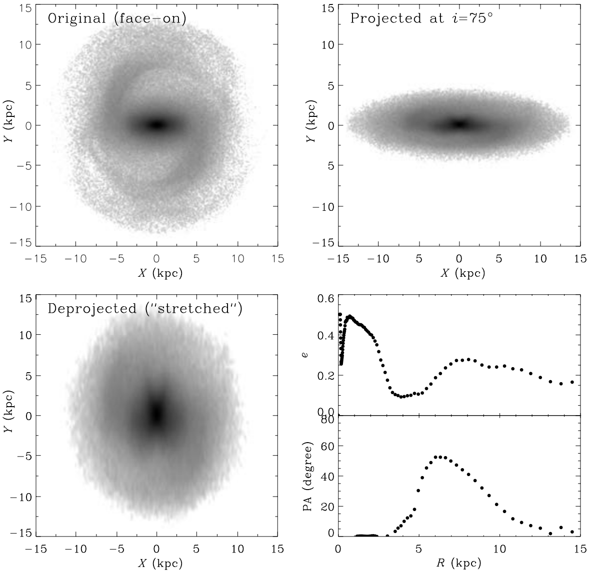
The problem obviously becomes much worse for non-axisymmetric galaxies.

Need an algorithm for systematically exploring the range of possible 3d shapes and orientations consistent with the observed photometry.

Some work has been done earlier but is not widely known...

[Romanowsky & Kochanek 1997; Magorrian 1999; Chakrabarty 2010]

IMFIT [Erwin 2015] can use 3d density profiles and compute projections during image fitting.



[Zou, Shen & Li 2014]

Example of artifacts in bar deprojection

Uncertainties and degeneracies in determining the potential

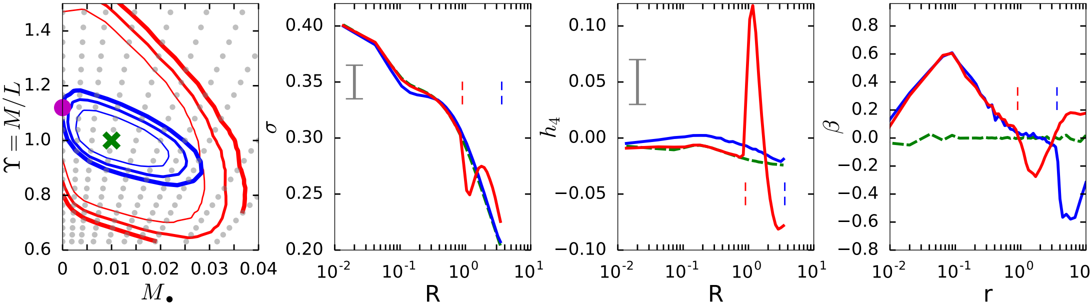
measure — want to infer

$$\mathcal{F}(X,\ Y,\ v_{\rm los}) \Longrightarrow \begin{cases} f(E, I_2, I_3) \\ \Phi(x, y, z) \end{cases}$$

Expect a large range of possible potentials consistent with observed kinematics!

Constraints get tighter when increasing spatial coverage (by excluding unrealistic orbit distributions) *or* having imperfect/noisy data

(see discussions in Dejonghe&Merritt 1992; Valluri+ 2004; Magorrian 2006, 2013).



Mock data: spherical Hernquist model with a black hole

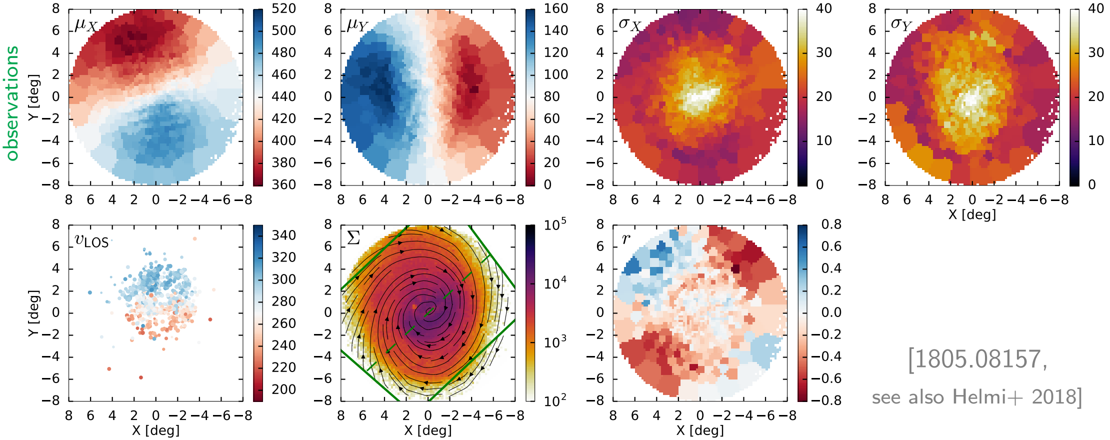
Troubles with statistical foundations of the method

Schwarzschild's method is extremely flexible, *but* like any non-parametric approach, is prone to overfitting and full of degeneracies.

- Distribution of orbit weights in best-fit models may be extremely jagged $\Longrightarrow$ need to regularize the ill-conditioned inverse problem to obtain physically meaningful solutions [e.g., Merritt 1993; Valluri+ 2004]
- Best-fit χ^2 is subject to discreteness noise in orbit space $\Longrightarrow$ scatter in χ^2 values for nearby models can be $\gg 1$
- Picking up just one possible DF for the given potential ignores the fact that many rather different orbit combinations produce similar χ^2 values $\Longrightarrow$ need to marginalize over [all possible?] DFs

  [Magorrian 2006, 2013; Bovy+ 2018; Prashin's talk]
- Confidence intervals on model parameters determined by $\Delta\chi^2 \lesssim \mathcal{O}(1)$ are unrealistically narrow; no reasons to expect that likelihood follows χ^2 statistics in the presence of $\mathcal{O}(10^4 - 10^5)$ hidden parameters (orbit weights)

Future directions: fitting 3d velocities

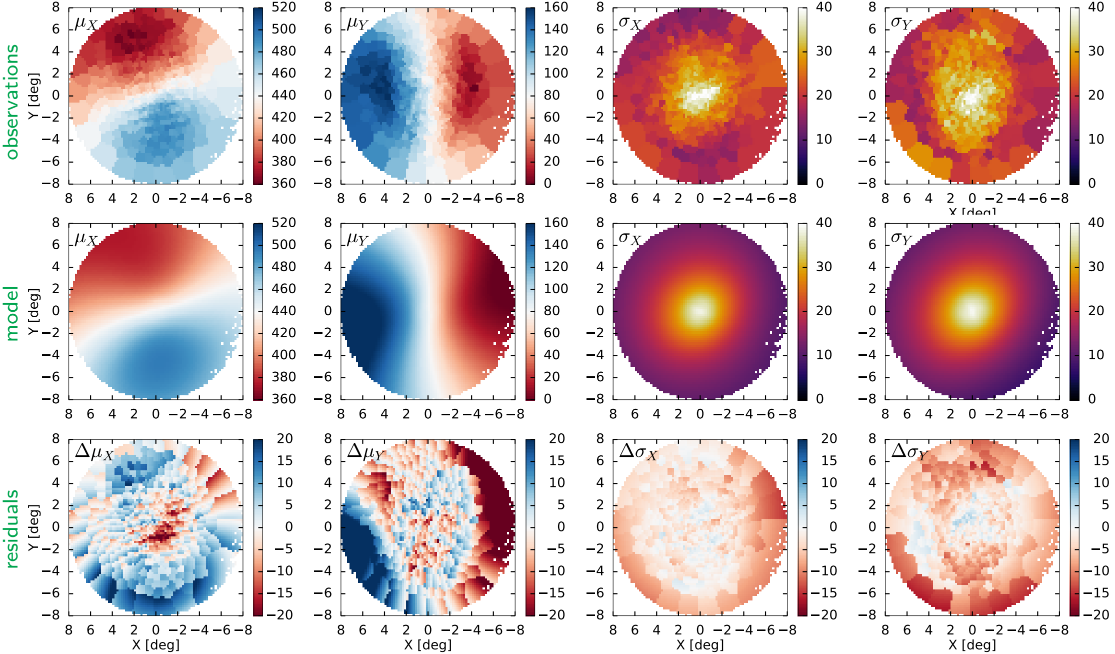


[1805.08157, see also Helmi+ 2018]

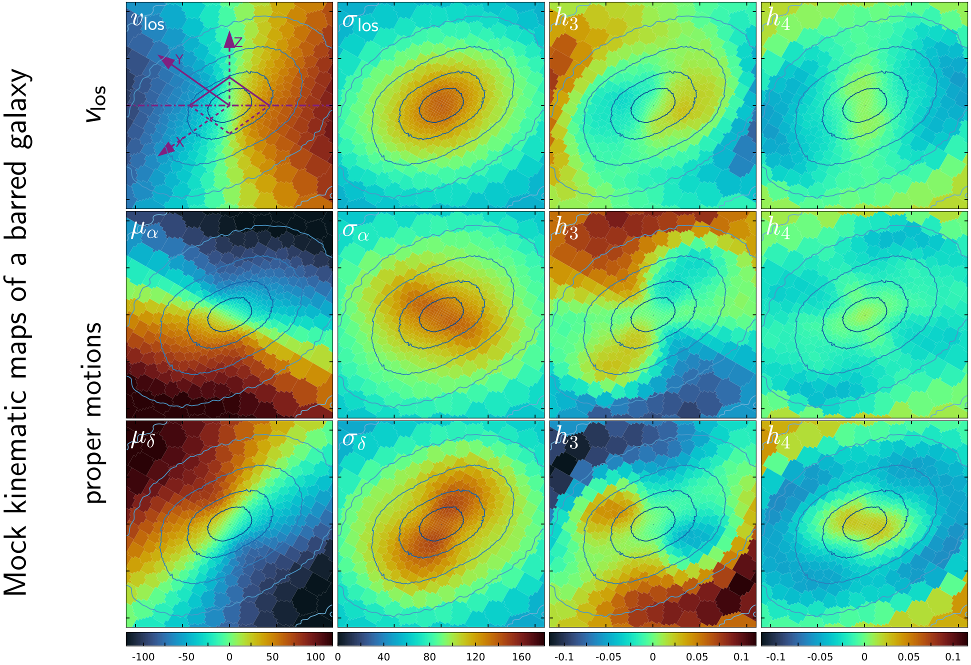
Proper motion maps of the Large Magellanic Cloud from Gaia DR2

Residuals from best-fit axisymmetric models might point to bar-induced kinematics...
...or just be artifacts from Gaia systematic errors – need to wait for DR3 to confirm

Future directions: fitting 3d velocities



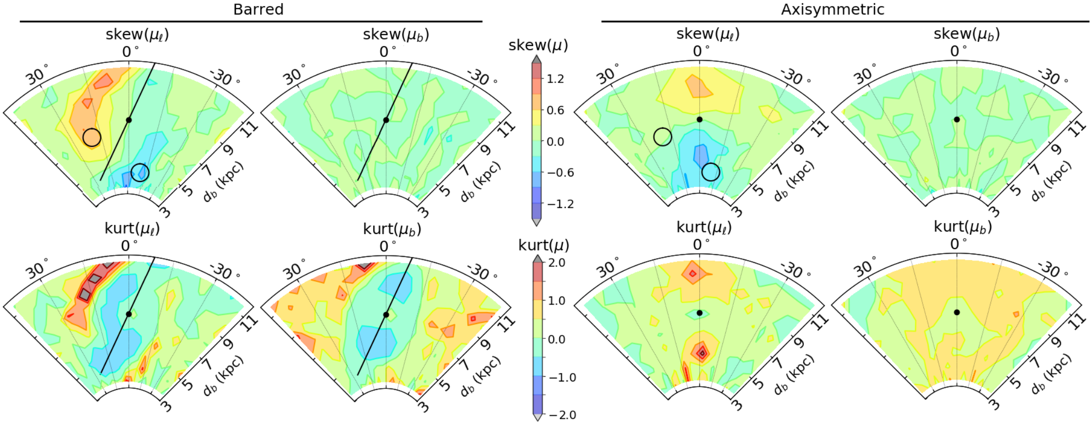
Future directions: fitting 3d velocities



Future directions: fitting 3d velocities

Milky Way bar now observed both in $v_{\rm los}$ and proper motions μ_α, μ_δ;
even a limited distance information can be inferred from magnitudes of red clump stars.

Bar pattern speed measured from M2M models [Portail+ 2017, Clarke+ 2019] and generalized Tremaine–Weinberg method [Sanders+ 2019].



predictions for Milky Way bar kinematics in Gaia [Palicio+ 2020]

Summary

Schwarzschild's method is a useful tool for dynamical modelling,

but... a number of open questions remain:

- Deprojection of complex morphology galaxies
- Intrinsic degeneracies in potential determination
- Large number of degrees of freedom
- Statistically sound confidence intervals
- Biases in black hole mass measurements

The tools are available for the community!