Dynamical modelling of stellar systems in the Gaia era

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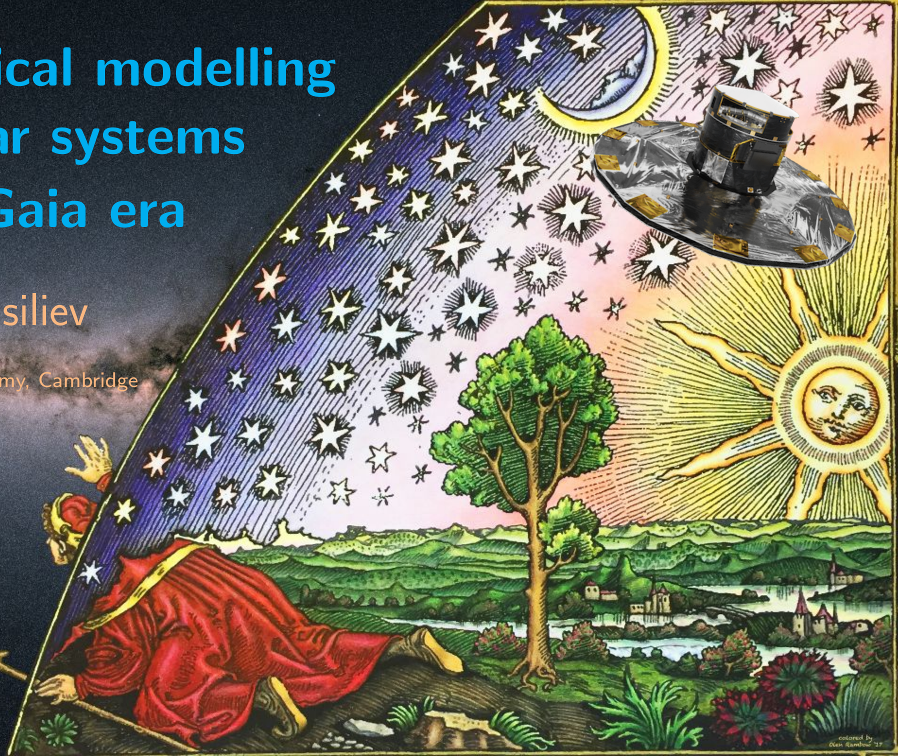
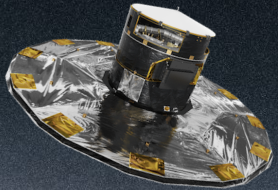


Synopsis



Overview of dynamical modelling

Overview of the Gaia mission

Examples:

Large Magellanic Cloud
Globular clusters
Measurement of the Milky Way gravitational potential

What does “dynamical modelling” mean?

It does **not** refer to a simulation (e.g. N-body) of the evolution of a stellar system.

Most often, it means “modelling a stellar system in a dynamical equilibrium” (used interchangeably with “steady state”).

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Fred Hoyle

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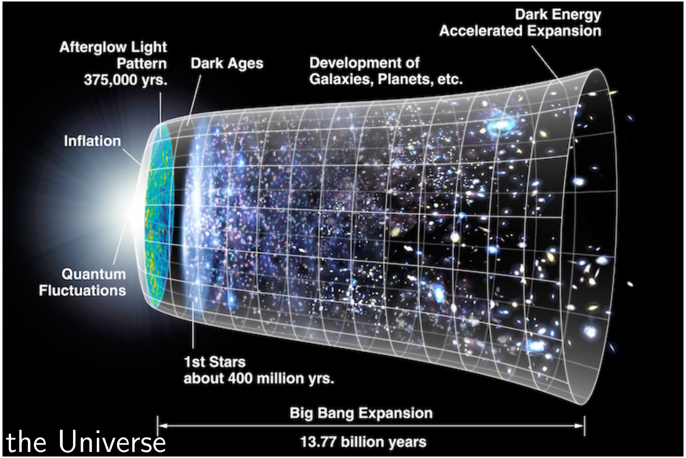
vs.



Why steady state?

Distribution function of stars $f(\mathbf{x}, \mathbf{v}, t)$
satisfies [sometimes] the collisionless Boltzmann equation:

$$\frac{\partial f(\mathbf{x}, \mathbf{v}, t)}{\partial t} + \mathbf{v} \frac{\partial f(\mathbf{x}, \mathbf{v}, t)}{\partial \mathbf{x}} - \frac{\partial \Phi(\mathbf{x}, t)}{\partial \mathbf{x}} \frac{\partial f(\mathbf{x}, \mathbf{v}, t)}{\partial \mathbf{v}} = 0.$$

Potential $\Leftrightarrow$ mass distribution

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Steady-state assumption $\Longrightarrow$ Jeans theorem:

$$f(\mathbf{x},\mathbf{v}) = f\left(\mathcal{I}(\mathbf{x},\mathbf{v};\ \Phi)\right)$$

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3D

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observations: 3D – 6D

integrals of motion ($\leq$ 3D?), e.g., $\mathcal{I} = \{E, L, \ldots\}$

Flavours of dynamical modelling

Tracers	individual stars	integrated light
Photometry	star counts	surface brightness profile
Kinematics	line-of-sight velocities ($v_{\rm LOS}$), proper motions (PM)	LOS velocity distribution $f(v_{\rm LOS})$, $\overline{v}, \sigma$, Gauss–Hermite moments

Methods Jeans eqns; distribution functions $f(\mathcal{I})$; orbit superposition

Dynamical self-consistency?

$$\nabla^2\Phi(\mathbf{x}) = 4\pi\, G\, \rho(\mathbf{x}) = 4\pi\, G \iiint f(\mathbf{x},\mathbf{v})\, d^3v$$

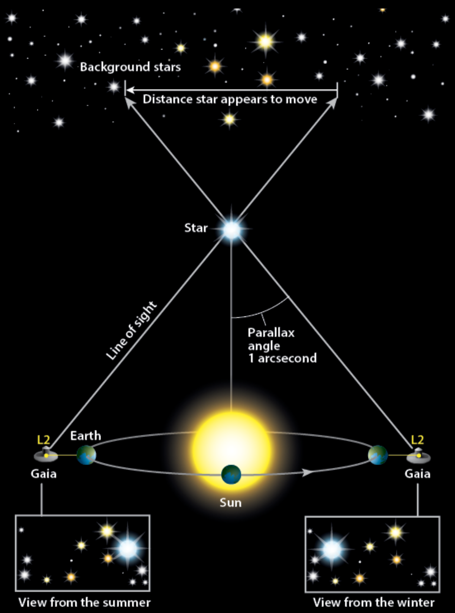
Outcome total potential Φ; dark matter distribution; internal kinematics

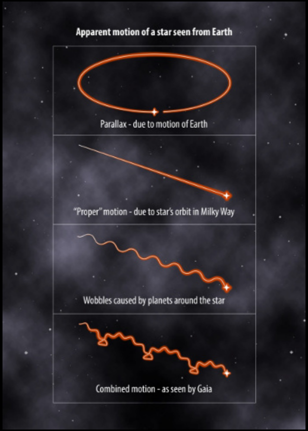
AGAMA – All-purpose galaxy modeling architecture

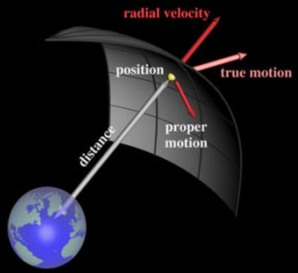
- Extensive collection of gravitational potential models
  (analytic profiles, azimuthal- and spherical-harmonic expansions)
  constructed from smooth density profiles or N-body snapshots;
- Conversion to/from action/angle variables;
- Self-consistent multicomponent models with action-based DFs;
- Schwarzschild orbit-superposition models;
- Generation of initial conditions for N-body simulations;
- Various math tools: 1d,2d,3d spline interpolation, penalized spline fitting and density estimation, multidimensional sampling;
- Efficient and carefully designed C++ implementation, examples, Python and Fortran interfaces, plugins for Galpy, NEMO, AMUSE.

`arXiv:1802.08239`, `1802.08255`, `1912.04288`; `http://agama.software`

Astrometry 101







Position on the sky α, δ

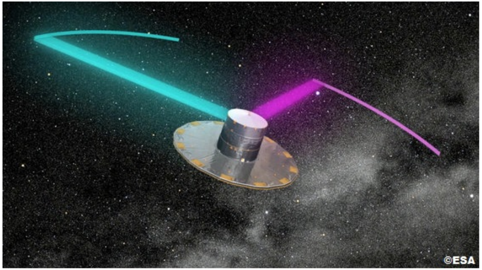
Parallax $\varpi = 1/\text{distance}$

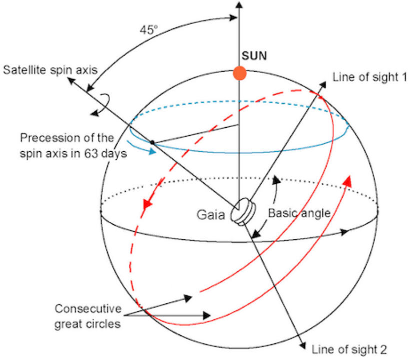
Proper motion μ_α, μ_δ

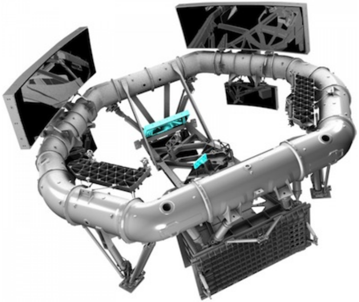
Line-of-sight velocity v_{los}

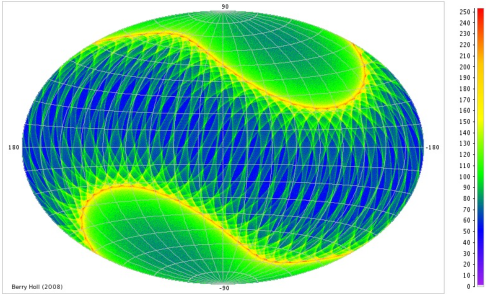
Binary orbit parameters

How Gaia astrometry works



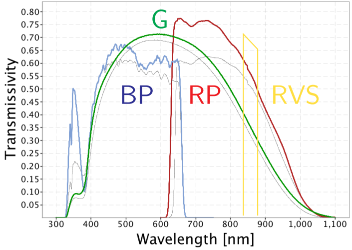


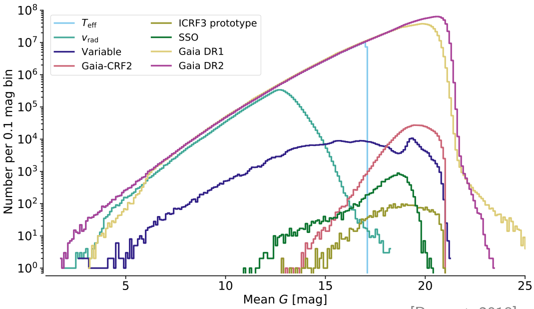




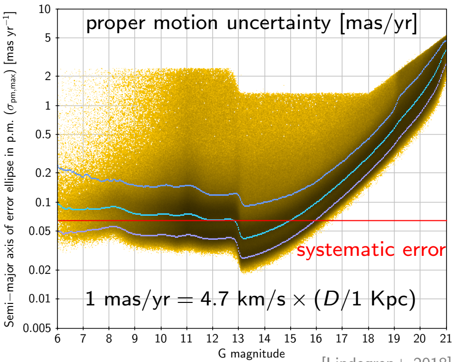
Overview of Gaia Data Release 2

- Scanning the entire sky every couple of weeks
- DR2 based on 22 months of observations
- Astrometry for sources down to 21 mag (1.3×10^9)
- Broad-band blue/red photometry (1.4×10^9)
- Radial velocity down to ~ 13 mag ($\sim 7 \times 10^6$ stars)
- ! No special treatment of binary stars
- ! Poor completeness in crowded fields



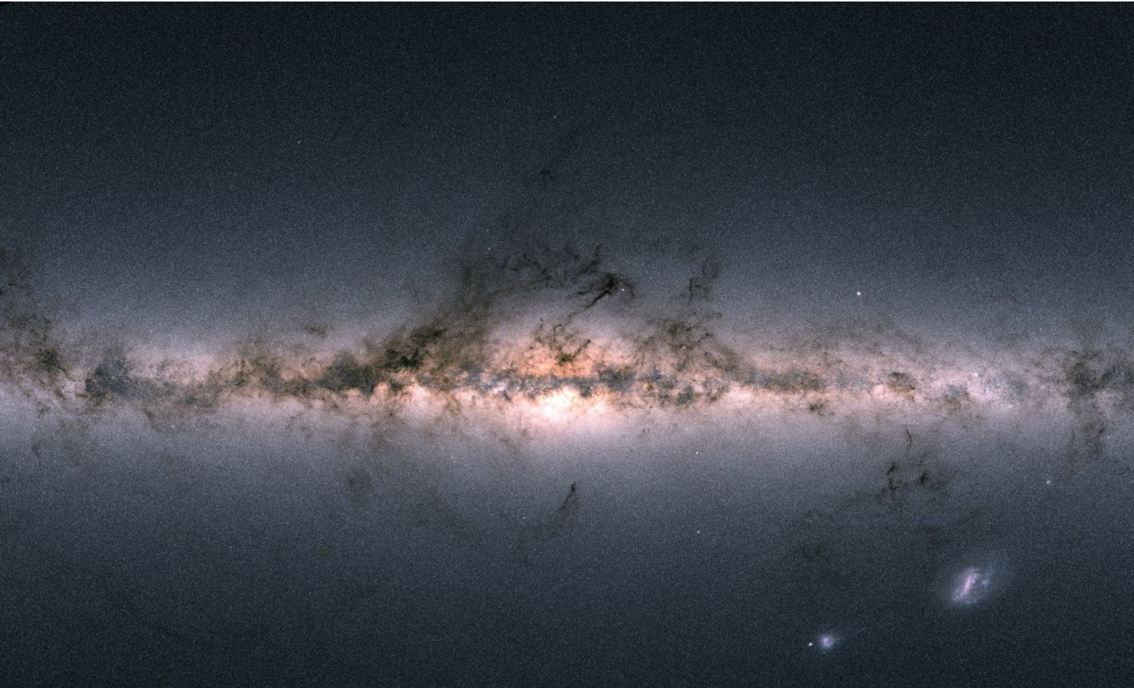


[Brown+ 2018]



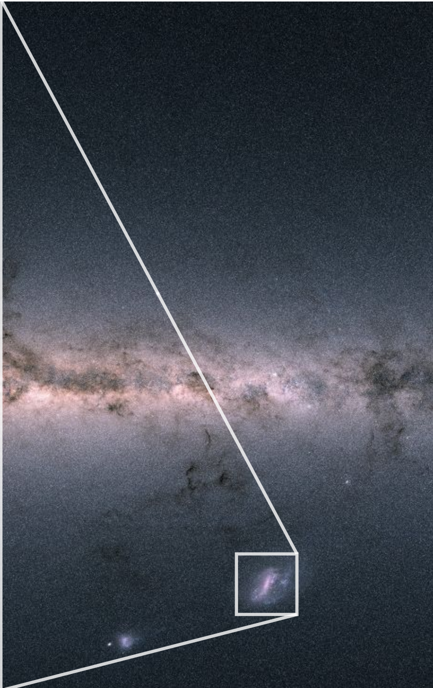
[Lindegren+ 2018]

Example 1: Large Magellanic Cloud

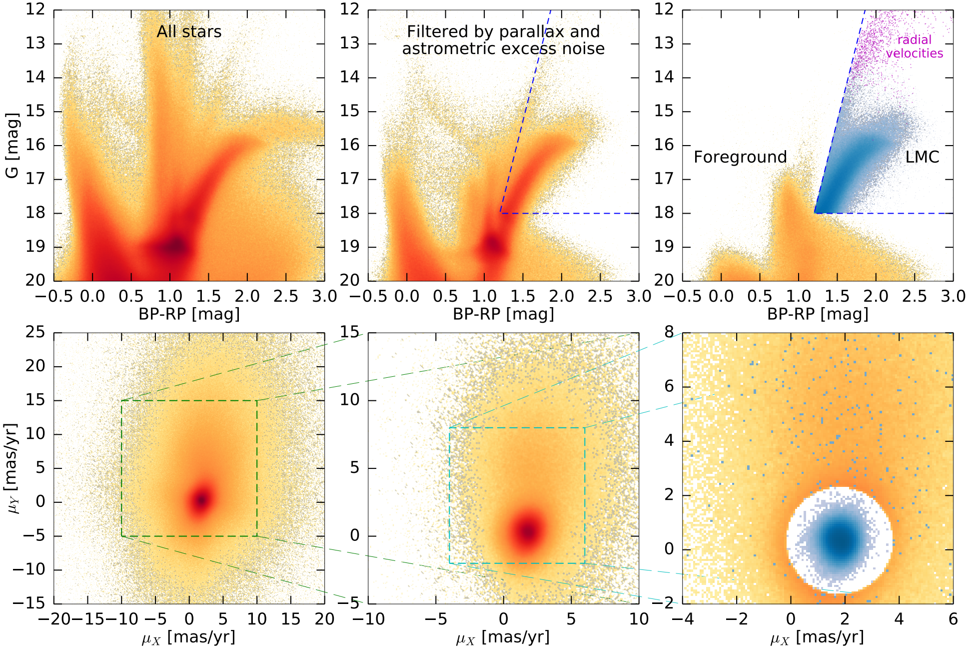


Example 1: Large Magellanic Cloud

[credit: ESA/Gaia/DPAC, 25/04/2018]



Gaia DR2 stars in the Large Magellanic Cloud



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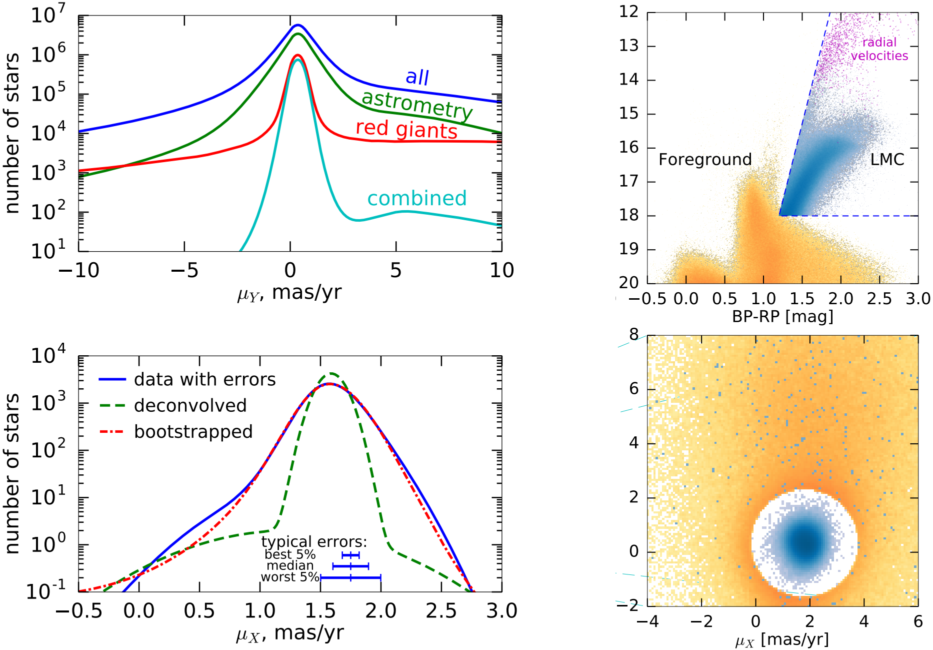
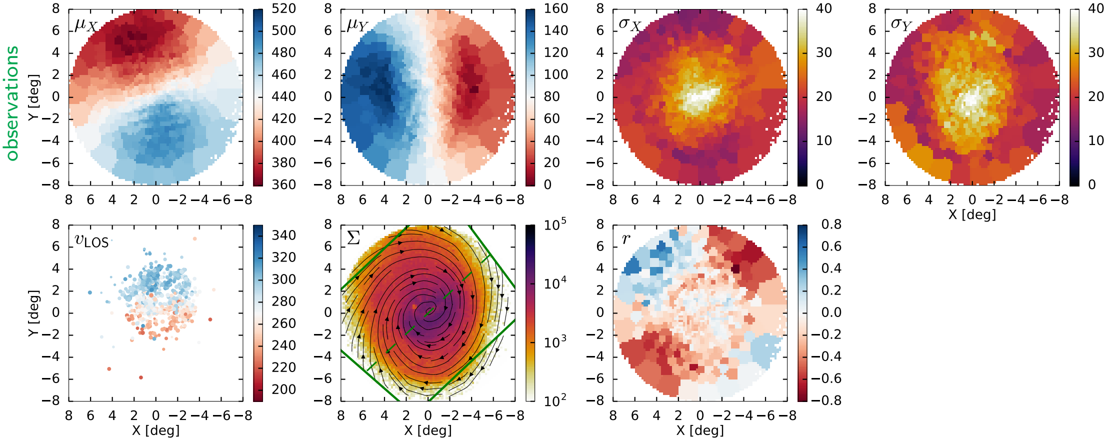


Internal kinematics of the Large Magellanic Cloud



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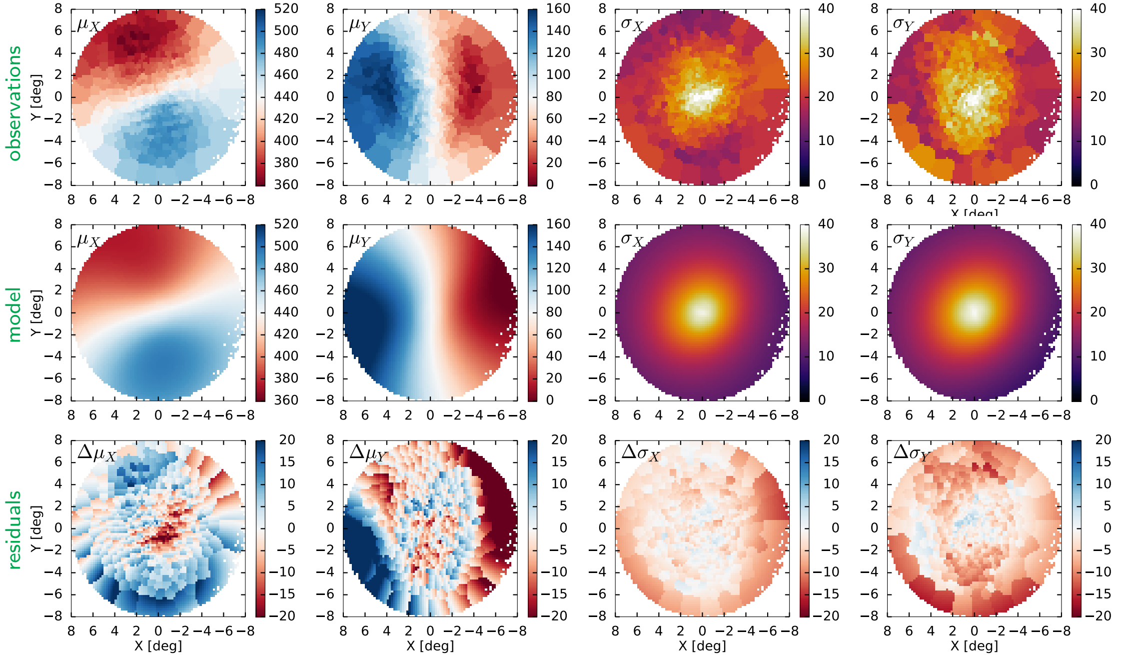
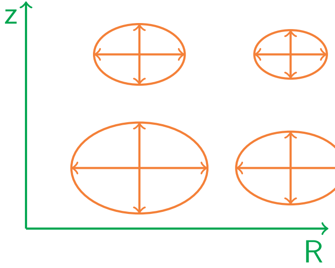


Internal dynamics of the Large Magellanic Cloud

[1805.08157]

Method: axisymmetric Jeans Anisotropic model
(a generalization of Cappellari 2002).



Assume that the velocity ellipsoid is aligned with cylindrical coordinates and has an axis ratio $\sigma_R^2/\sigma_z^2 = b =$ const.

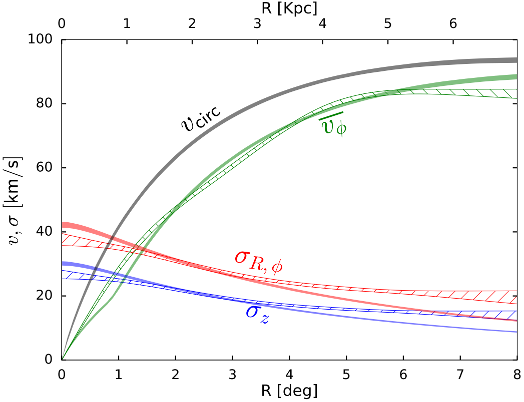
Total gravitational potential:
$\Phi(R,z) = \Phi_{\rm stars} + \Phi_{\rm DM\ halo}$.

Density of tracers (stars):
$\rho(R,z) = \frac{1}{4\pi G}\nabla^2\,\Phi_{\rm stars}$.

Compute $\sigma_{R,z}$ from Jeans eqns:

$$0 = \rho\,\frac{\partial\Phi}{\partial z} + \frac{\partial(\rho\,\sigma_R^2)}{\partial z}\,\frac{1}{b}$$

$$0 = \rho\,\frac{\partial\Phi}{\partial R} + \frac{\partial(\rho\,\sigma_R^2)}{\partial R} + \frac{\rho\,\left(\sigma_R^2 - \overline{v_\phi^2}\right)}{R}$$

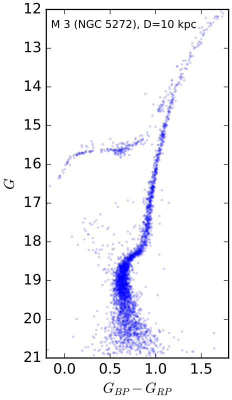


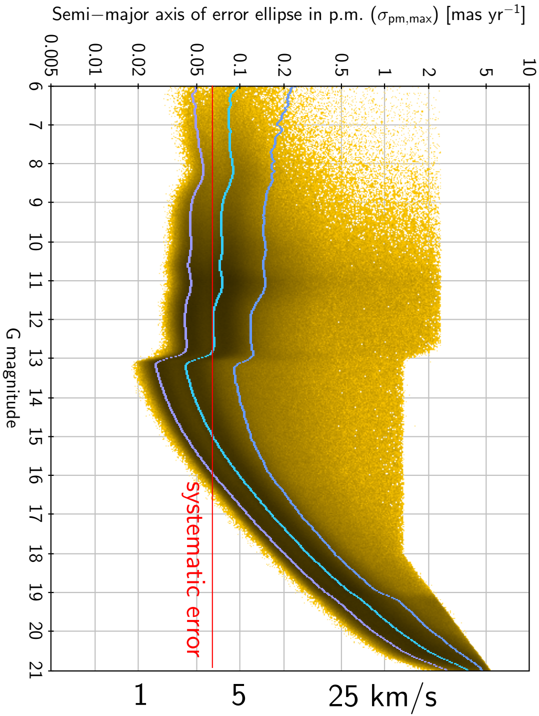
Example 2: globular clusters

Challenges:

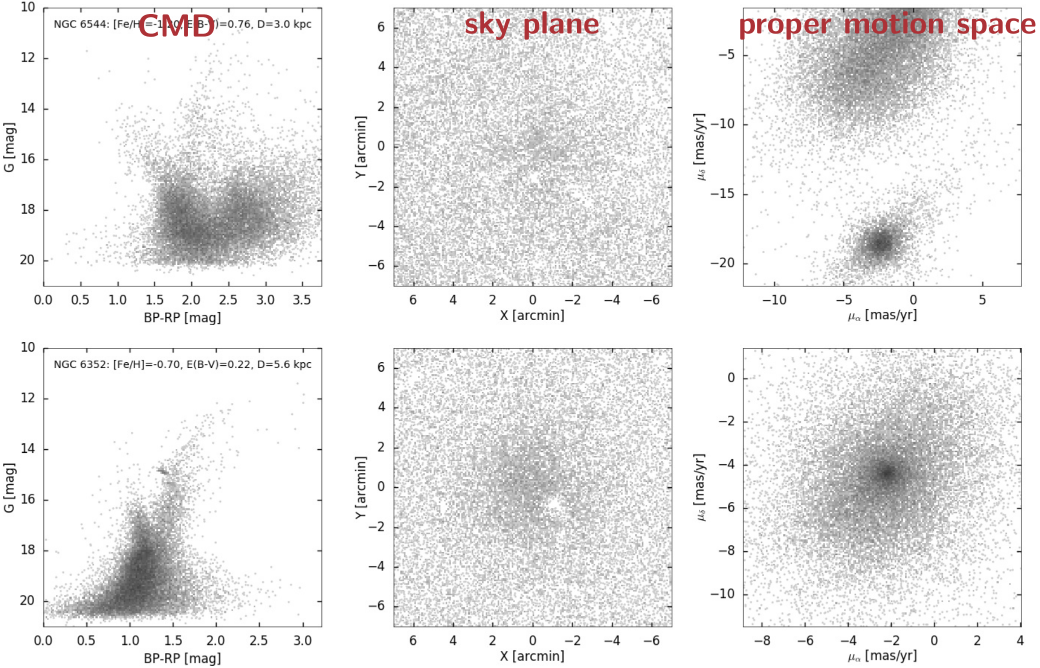
- crowding (can't study central regions)
- contamination from galactic stars
- systematic error is of order σ

CMD of a typical cluster at 10 kpc





Determination of cluster membership



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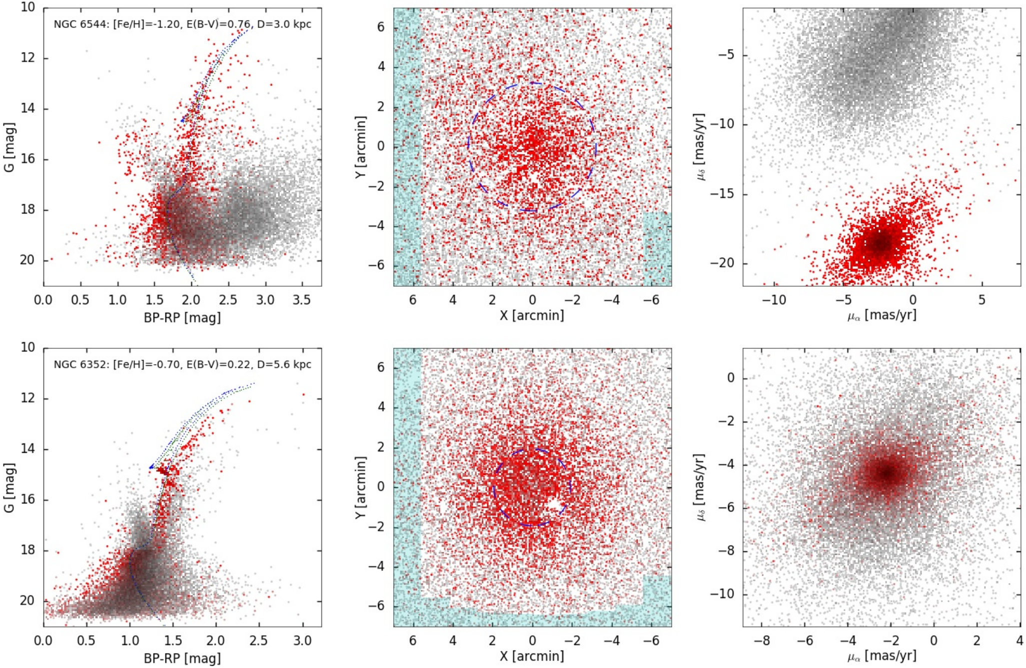
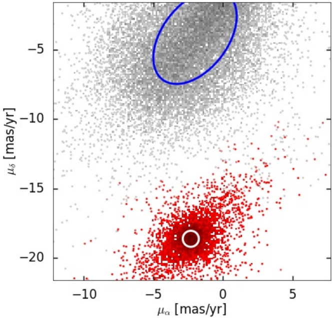


Probabilistic membership determination



A hard cutoff in PM space is not always possible and is conceptually unsatisfactory.

A more mathematically well-grounded alternative: gaussian mixture modelling.

$$f(\boldsymbol{\mu}_i) = q\,\mathcal{N}(\boldsymbol{\mu}_i \mid \overline{\boldsymbol{\mu}_{\rm cl}}, \Sigma_{{\rm cl};i}) + (1-q)\,\mathcal{N}(\boldsymbol{\mu}_i \mid \overline{\boldsymbol{\mu}_{\rm fg}}, \Sigma_{{\rm fg};i})$$

$$\mathcal{N}(\boldsymbol{\mu} \mid \overline{\boldsymbol{\mu}}, \Sigma) \equiv \frac{\exp\left[-\frac{1}{2}(\boldsymbol{\mu}-\overline{\boldsymbol{\mu}})^T\,\Sigma^{-1}\,(\boldsymbol{\mu}-\overline{\boldsymbol{\mu}})\right]}{2\pi\,\sqrt{\det\Sigma}},$$

where the mean PMs $\overline{\boldsymbol{\mu}}$ and dispersions Σ of the cluster and foreground distributions, and the fraction of cluster members q, are all inferred by maximizing the likelihood of the observed stellar PMs $\ln\mathcal{L} \equiv \sum_{i=1}^{N_{\rm stars}} \ln f(\boldsymbol{\mu}_i)$.

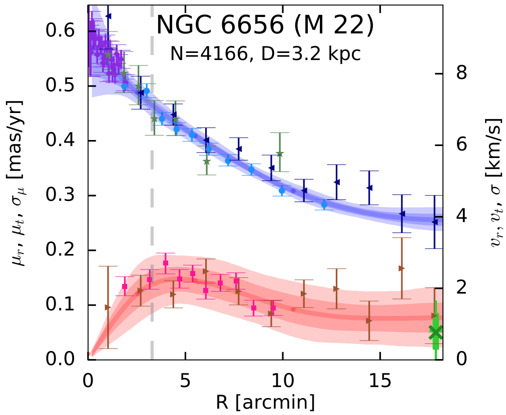
Posterior membership probability for each star:

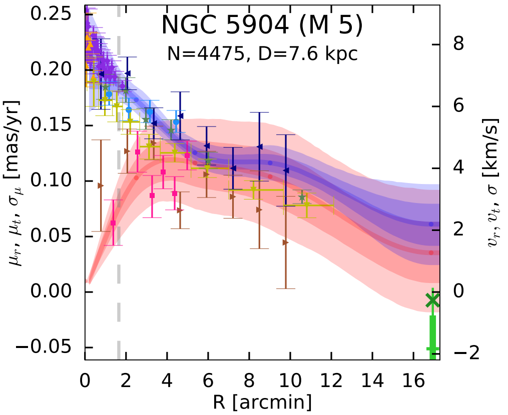
$$p_{{\rm cl};i} = \frac{q_{\rm cl}(\mathbf{r}_i)\,\mathcal{N}(\boldsymbol{\mu}_i \mid \overline{\boldsymbol{\mu}_{\rm cl}}, \Sigma_{{\rm cl};i})}{q_{\rm cl}(\mathbf{r}_i)\,\mathcal{N}(\boldsymbol{\mu}_i \mid \overline{\boldsymbol{\mu}_{\rm cl}}, \Sigma_{{\rm cl};i}) \;+\; [1-q_{\rm cl}(\mathbf{r}_i)]\,\mathcal{N}(\boldsymbol{\mu}_i \mid \overline{\boldsymbol{\mu}_{\rm fg}}, \Sigma_{{\rm fg};i})}$$

Internal kinematics of globular clusters

Clear signature of rotation in ~ 10 clusters:

see also Bianchini+ 2018, Sollima+ 2019, Jindal+ 2019, all based on Gaia data



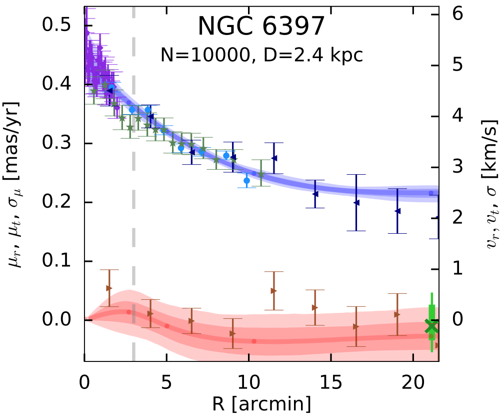


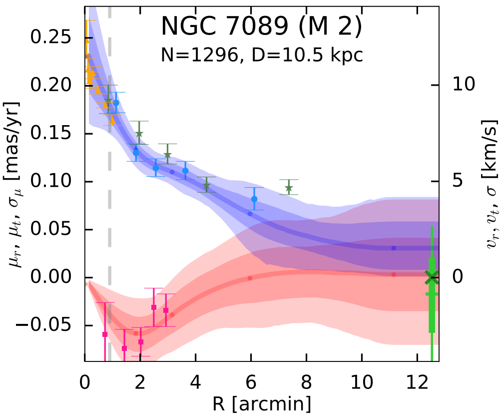
Internal kinematics of globular clusters

[1811.05345]

Good match between line-of-sight velocity dispersion and PM dispersion:

see also Baumgardt+ 2018, Jindal+ 2019 (Gaia PM) and
HST measurements in central parts [Bellini+ 2014, Watkins+ 2015]

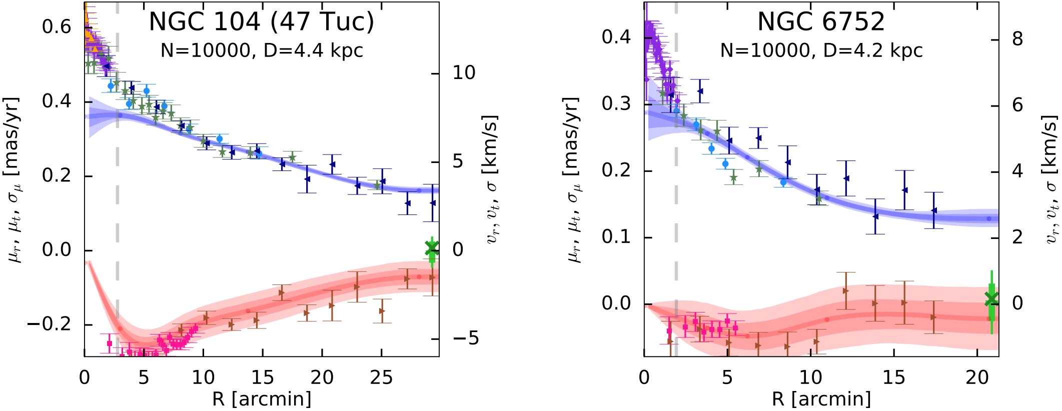




Internal kinematics of globular clusters

Mismatch between $\sigma_{\rm los}$ and PM dispersion in central parts:

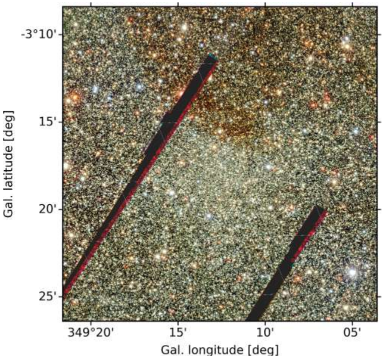
likely due to crowding issues and aggressive sample cleanup



The globular cluster FSR 1758

"Sequoia in the forest" recently discovered by Barbá+2019 in the DECaPS legacy survey of the Galactic bulge [Schlafly+2018].

Despite being located in a highly crowded region, it stands out very clearly in Gaia PM.



[Barbá+2019]

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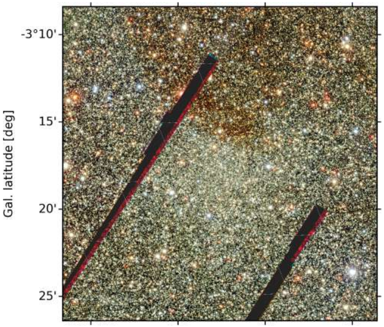
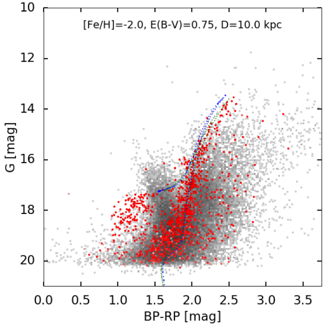
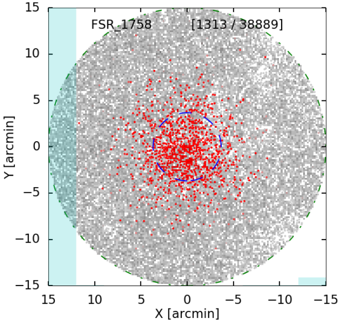
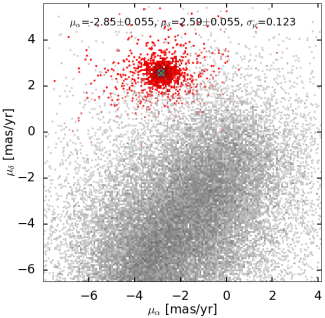


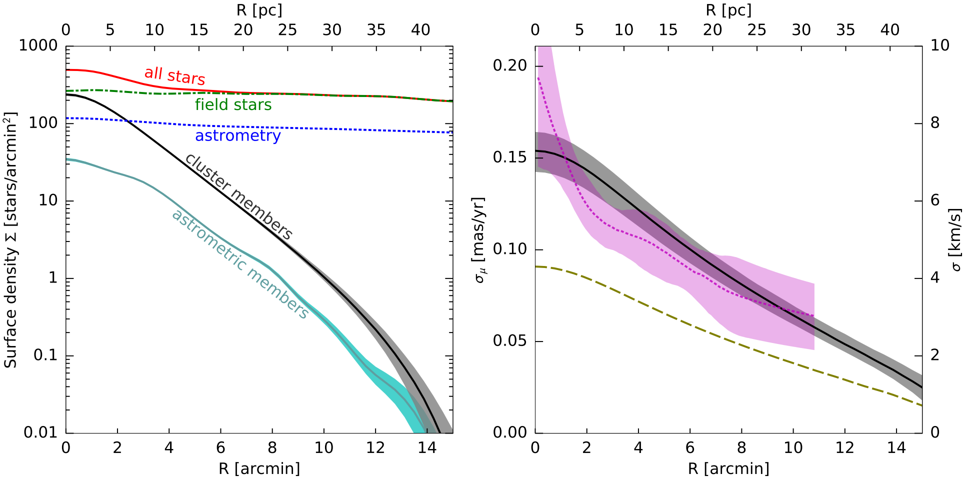






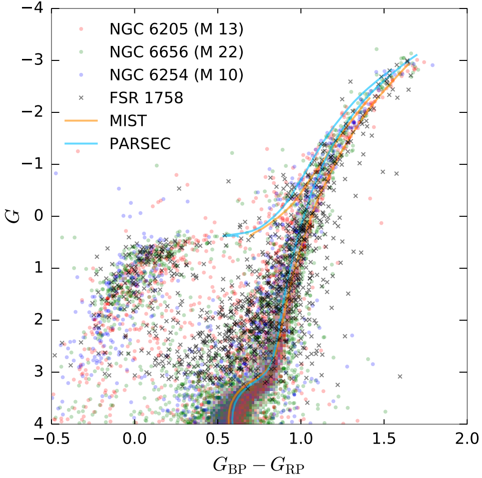
Dynamical modelling of FSR 1758

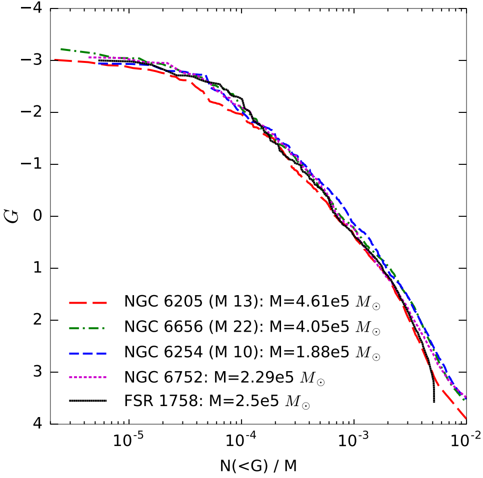
Fit a mixture model to the spatial and PM distribution of the cluster and field stars, using a generalized King model for the cluster density and PM dispersion profiles:



Photometric modelling of FSR 1758

Compare the CMD of member stars with that of several analogous globular clusters, determine the total mass by scaling the magnitude distribution:



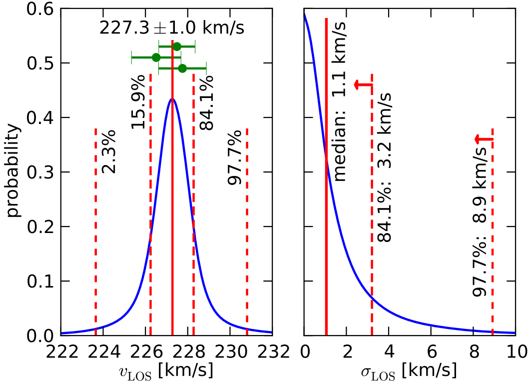


Line-of-sight velocity dispersion of FSR 1758

The brightest stars happened to be recorded by Gaia RVS;

Simpson 2019 used them to determine v_{LOS} and the orbit of the cluster, which turned out to be strongly retrograde.

σ_{LOS} from 3 stars (!) in Gaia RVS



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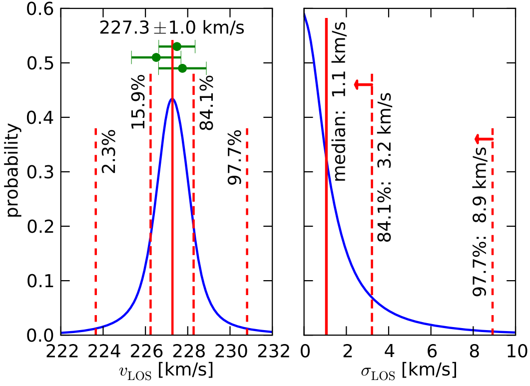
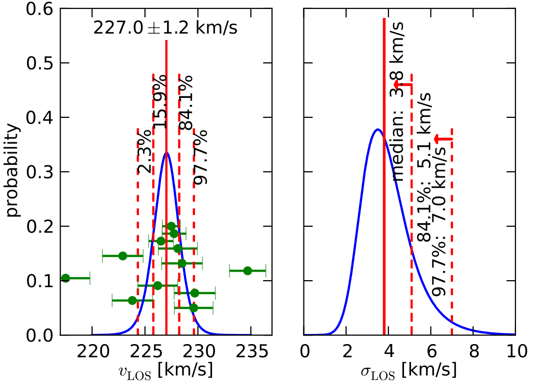
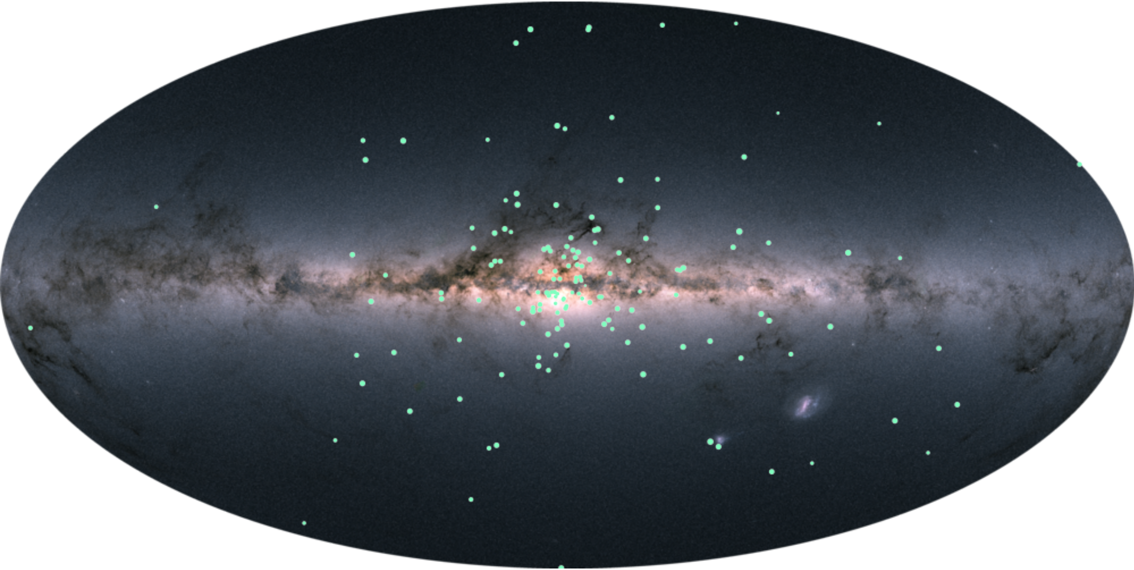


$\sigma_{\rm LOS}$ with additional 9 stars from ground-based spectroscopy [Villanova+2019]



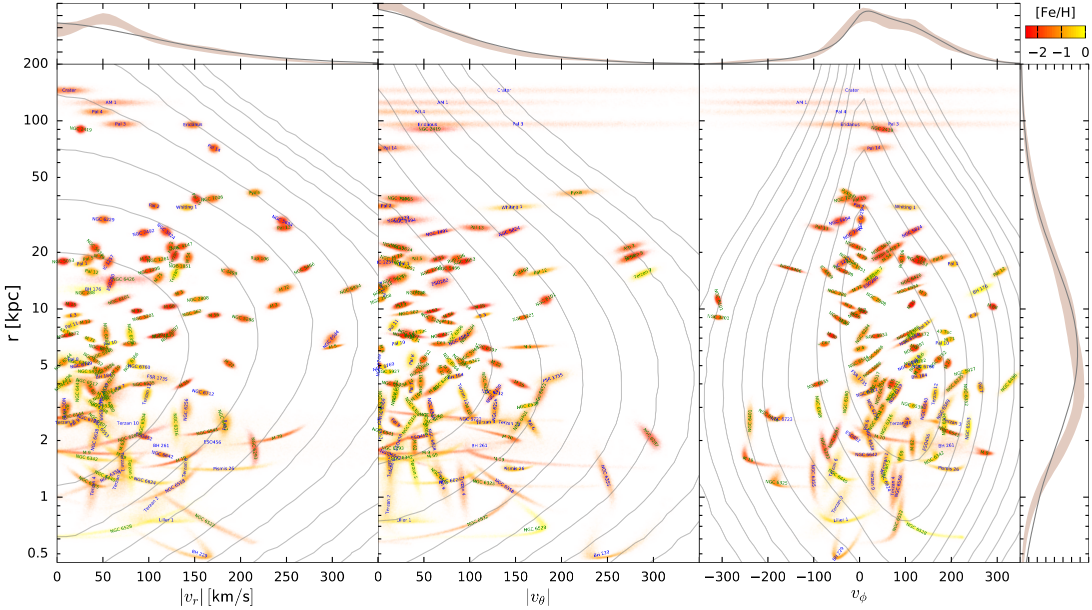
Example 3: the entire Milky Way globular cluster system

1807.09775, see also Gaia Collaboration (Helmi+) 2018, Baumgardt+ 2018, de Boer+ 2019



$\sim$ 150 globular clusters in the Milky Way

Distribution of globular clusters in position/velocity space



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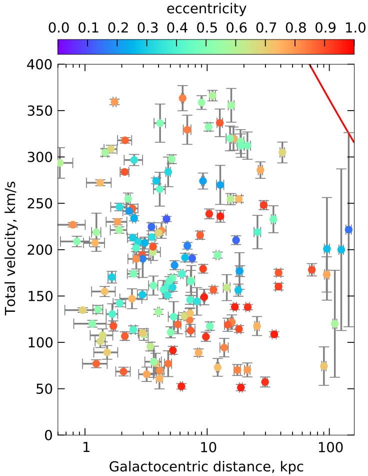
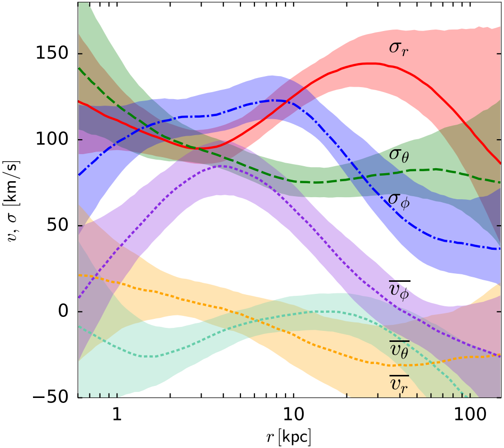
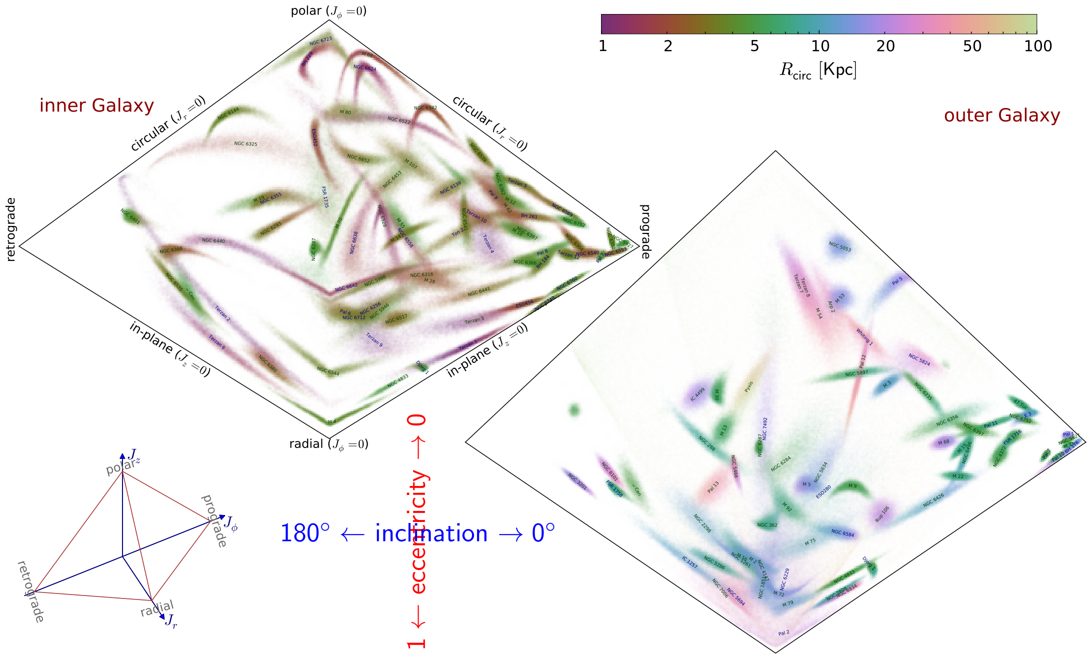




- Significant overall rotation, especially within the central 10 kpc.
- Nearly isotropic dispersion at $r < 10$ kpc, more radially anisotropic in outer parts.
- A population of ~ 10 clusters on eccentric orbits [Myeong+ 2018].
- Correlated orbits (e.g., Sgr stream: M 54, Terzan 7, Terzan 8, Arp 2, Pal 12, Whiting 1).

Distribution of globular clusters in action space



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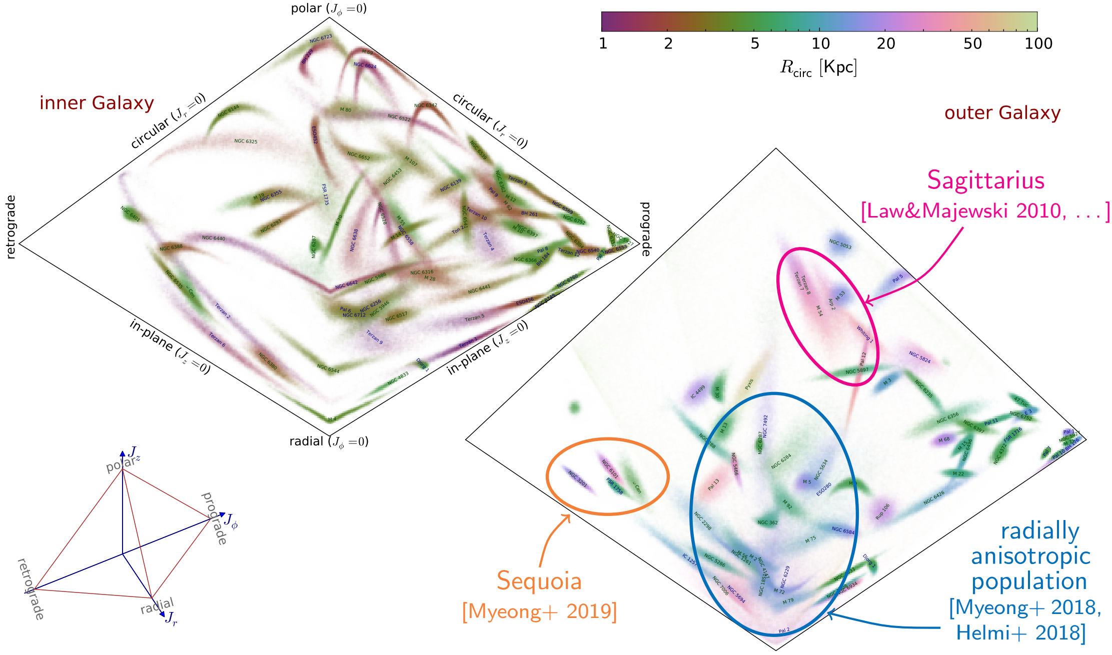


Distribution of globular clusters in action space

Jackson Pollock, "Convergence"

Kliment Redko, "Uprising"

Dynamical modelling of the entire globular cluster population

Assume a steady-state equilibrium model for the Milky Way globular cluster distribution function (DF): $f\left(\mathbf{J} \mid \boldsymbol{\alpha}\right)$;

actions: $\mathbf{J}\left(\mathbf{x}, \mathbf{v} \mid \Phi\right)$

potential: $\Phi(\mathbf{x} \mid \boldsymbol{\beta})$

free parameters

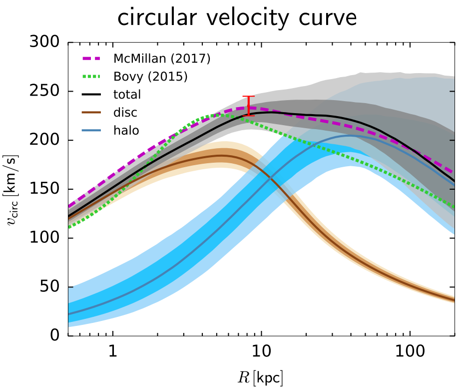
Maximize the likelihood of drawing the observed positions and velocities of all clusters (marginalizing over measurement uncertainties) by varying the free parameters $\boldsymbol{\alpha}, \boldsymbol{\beta}$ of the DF and the potential.

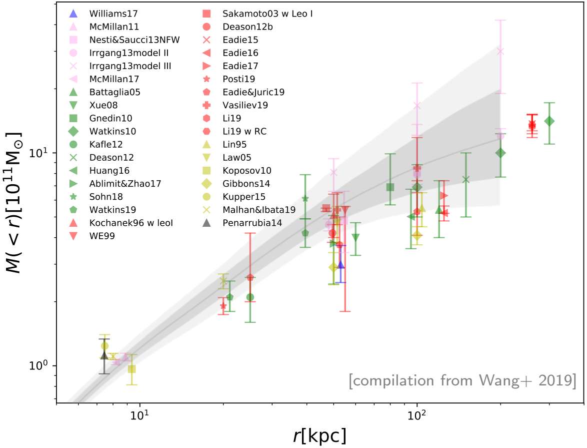
$$\max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ln \mathcal{L} \equiv \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \sum_{i=1}^{N_{\text{clusters}}} \ln \int f\left(\mathbf{J}(\mathbf{w} \mid \Phi)\right) \mathcal{N}(\mathbf{w} \mid \mathbf{w}_i, \delta\mathbf{w}_i)\, \mathrm{d}\mathbf{w}, \quad \mathbf{w} \equiv \{\mathbf{x}, \mathbf{v}\}$$

Explore the parameter space via MCMC and derive confidence intervals for $\boldsymbol{\alpha}, \boldsymbol{\beta}$.

Constraining the Milky Way potential

[1807.09775]





- Results are broadly consistent with other studies based on globular clusters [Binney&Wong 2017, Sohn+ 2018, Watkins+ 2019, Posti&Helmi 2019, Eadie&Juric 2019];
- Mass model from McMillan(2017) is acceptable, the one from Bovy(2015) is too light.

Summary

Gaia is a treasure cove for dynamical modelling!

Summer school in applied galactic dynamics

July 6 – August 16, 2020
Center for Computational Astrophysics / Flatiron Institute, New York City

Connecting dynamics theory and galactic observations, the Applied Galactic Dynamics School will consist of one week of lectures and tutorials, explorations of available data sets, and project discussions, and 5 weeks for individual projects. During the projects there will be regular group meetings and plenty of informal gatherings to foster cross-project connections, and to make optimal use of the expertise present at the School.

Support for students includes roundtrip airfare, a meal-plan and accommodation for all six weeks of the school. Students are expected to attend all six weeks.

Organizers, mentors and lecturers:

Emily Cunningham, Elena D'Onghia, Victor Debattista, Keith Hawkins, Amina Helmi, Jason Hunt, Kathryn Johnston, Melissa Ness, Sarah Pearson, Adrian Price-Whelan, Robyn Sanderson, David Spergel, Tjitske Starkenburg, Monica Valluri, Eugene Vasiliev, Martin Weinberg, Larry Widrow.

Application deadline: February 15; website: galacticdynamics.nyc