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Schwarzschild modeling of barred galaxies

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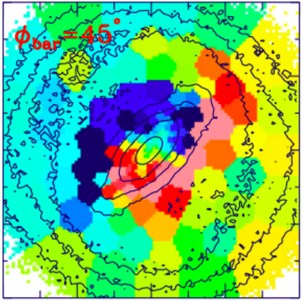
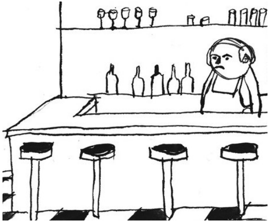
Motivation



- 146% galaxies in the Universe are barred
- Bars have specific kinematic signatures, especially in the higher-order Gauss–Hermite moments [e.g., Bureau & Athanassoula 2005; Debattista+ 2005; Méndez-Abreu+ 2014; Iannuzzi & Athanassoula 2015; Li+ 2018]
- Measurement of central supermassive black hole masses may be biased when ignoring a bar [e.g., Brown+ 2013]
- Bars are fun, so why not model them! [citation needed]



Challenges

- Complex morphology, complex kinematics $\Longrightarrow$ need sophisticated methods
- Various other caveats

Schwarzschild's orbit-superposition method: basics

Introduced by Schwarzschild(1979) as a practical approach
for constructing self-consistent triaxial models with prescribed $\rho(\mathbf{x}) \Leftrightarrow \Phi(\mathbf{x})$.

integrals of motion

To invert the equation $\rho(\mathbf{x}) = \iiint f\left(\mathcal{I}\left[\mathbf{x}, \mathbf{v} \mid \Phi\right]\right)\, d^3\mathbf{v}$,

discretize both the density profile and the distribution function:

$\rho(\mathbf{x}) \implies$ cells of a spatial grid; mass of each cell is $M_c = \iiint\limits_{\mathbf{x}\in V_c} \rho(\mathbf{x})\, d^3x$;

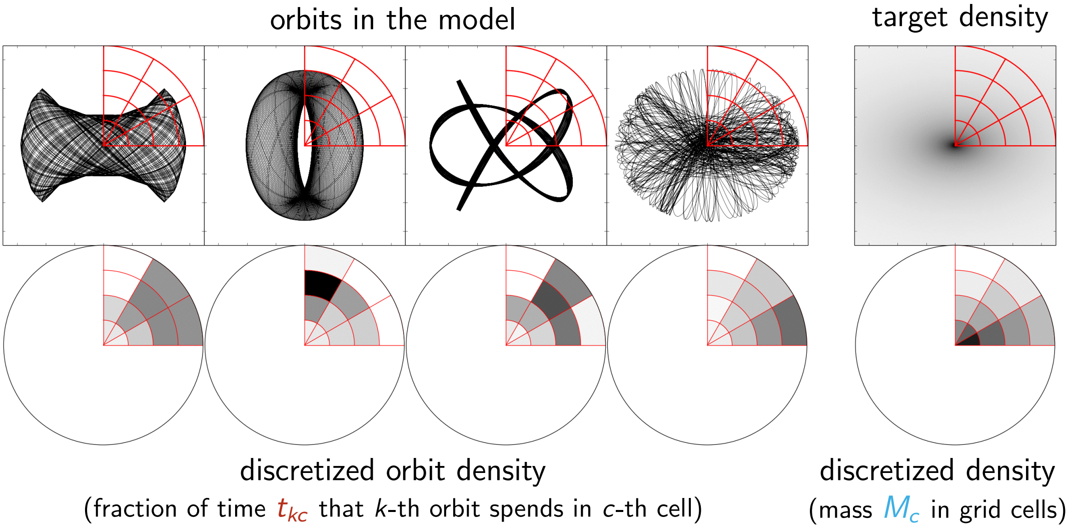
$f(\mathcal{I}) \implies$ collection of orbits with unknown weights [to be determined]:

$$f(\mathcal{I}) = \sum_{k=1}^{N_{\rm orb}} w_k\, \delta(\mathcal{I} - \mathcal{I}_k)$$

each orbit is a delta-function in the space of integrals of motion

adjustable weight of each orbit

Schwarzschild's orbit-superposition method: self-consistency



For each c-th cell we require $\sum_k w_k \, t_{kc} = M_c$, where $w_k \geq 0$ is orbit weight

Schwarzschild's orbit-superposition method: fitting procedure

- Assume some potential $\Phi(\mathbf{x})$
  (e.g., from the deprojected luminosity profile plus parametric DM halo or SMBH)
- Construct the orbit library in this potential:
  for each k-th orbit, store its contribution to the discretized density profile
  t_{kc}, $c = 1..N_\mathrm{cell}$ and to the kinematic observables u_{kn}, $n = 1..N_\mathrm{obs}$
- Solve the constrained optimization problem to find orbit weights w_k:

$$\text{minimize } \chi^2 + \mathcal{S} \equiv \sum_{n=1}^{N_\mathrm{obs}} \left(\frac{\sum_{k=1}^{N_\mathrm{orb}} w_k\, u_{kn} - U_n}{\delta U_n} \right)^2 + \mathcal{S}\big(\{w_k\}\big)$$

regularization term

observational constraints

their uncertainties

$$\text{subject to } w_k \ge 0, \quad k = 1..N_\mathrm{orb},$$

$$\sum_{k=1}^{N_\mathrm{orb}} w_k\, t_{kc} = M_c, \quad c = 1..N_\mathrm{cell}$$

density constraints (cell masses)

- Repeat for different choices of potential and find the one that has lowest χ^2

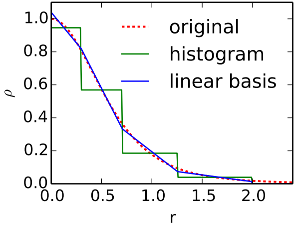
Schwarzschild's orbit-superposition method: implementations

Several commonly used independent implementations of the method:

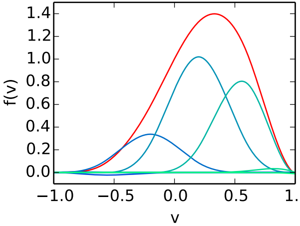
- theoretical studies in triaxial geometry: Schwarzschild 1979, 1993; Pfenniger 1984; Statler 1987; Merritt & Fridman 1996; Siopis & Kandrup 2000; Vasiliev 2013
- spherical codes: Richstone & Tremaine 1984; Rix+ 1997; Jalali & Tremaine 2010; Breddels & Helmi 2013; Kowalczyk+ 2017
- axisymmetric: "Leiden" [van der Marel, Cretton, Cappellari, . . . – since 1998]
- axisymmetric: "Nukers" [Gebhardt, Richstone, Kormendy, . . . – since 2000]
- axisymmetric: "MasMod" [Valluri, Merritt, Emsellem – since 2004]
- triaxial/Milky Way bar: Zhao, Wang, Mao 1996, 2012
- triaxial: van den Bosch, van de Ven, de Zeeuw, Zhu, . . . – since 2008
- triaxial: Vasiliev & Valluri, in prep.

New implementation of Schwarzschild's method: highlights

- arbitrary geometry (from spherical to triaxial), arbitrary density profiles ($\Rightarrow$ flexible Poisson solver)
- rotating frame ($\Rightarrow$ triaxial bars)
- random sampling of initial conditions for orbits
- several choices for 3d intrinsic density constraints (incl. piecewise-linear shape elements)



- representation of the 3d observational datacube (X, Y, v_{los}) in terms of B-splines
- either Gauss–Hermite moments or a full LOSVD fitting
- very efficient quadratic optimization solver



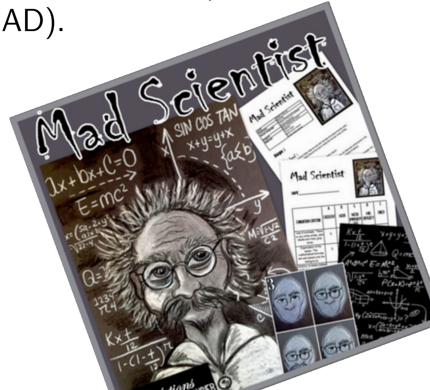
- Publicly available as part of AGAMA library for dynamical modelling

Measurement of gravitational potential

Simple spherical model: $\Phi(r) = \Upsilon\,\Phi_0(r) - G\,M_\bullet/r$, two free parameters: mass-to-light ratio Υ, SMBH mass $M_\bullet$.

Consider the line-of-sight through the galaxy center:
to fit a peak in velocity dispersion $\sigma_{\rm los}(R \approx 0)$, one may either increase $M_\bullet$, or make the orbits of stars more radially anisotropic.

Inference on the potential from $\sigma_{\rm los}(R)$ alone (e.g., in Jeans eqn) suffers from the mass–anisotropy degeneracy (MAD).



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Dejonghe & Merritt (1992) showed that one may uniquely infer $f(E,L)$ from the full LOSVD $\mathcal{F}(R, v_{\rm los})$ *in the given (assumed) potential*, and conjectured that the range of potentials for which $f(E,L) \geq 0$ is rather narrow.

$$\mathcal{F}(R, v_{\rm los}) \implies \begin{cases} f(E,L) \\ \Phi(r) \end{cases}$$

measure want to infer

more generally, $$\mathcal{F}(X, Y, v_{\rm los}) \implies \begin{cases} f(E, I_2, I_3) \\ \Phi(x,y,z) \end{cases}$$

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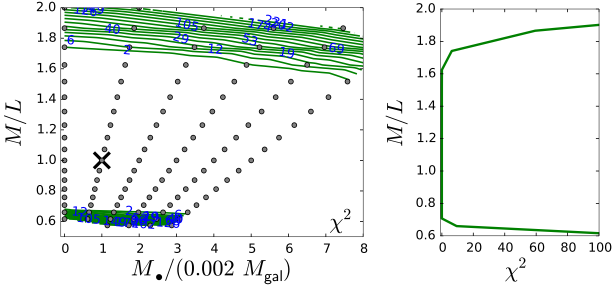
We can test this using a Gauss–Hermite representation of the LOSVD:

order of GH expansion: 6



A large range of potentials fit the [noiseless] LOSVD perfectly

[Valluri, Merritt & Emsellem 2004]

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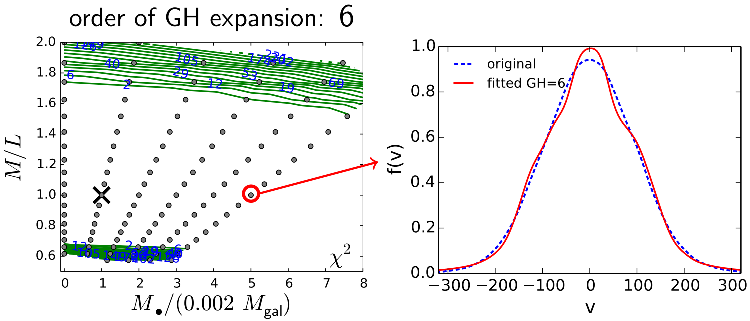

LOSVD of the model matches the first 6 GH moments, but is unrealistically wiggly

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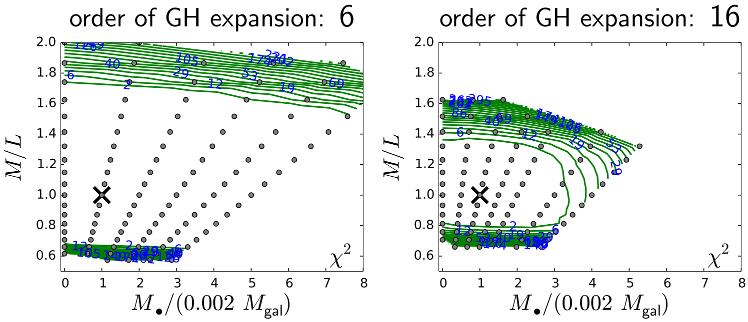
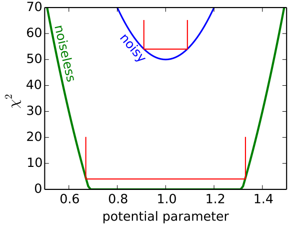


Still a rather broad range of possible Υ and especially $M_\bullet$!

Measurement of gravitational potential: degeneracies

Magorrian (2006) points out that this degeneracy only exists in the noise-free case, but disappears when the data is noisy and the fit is not perfect.

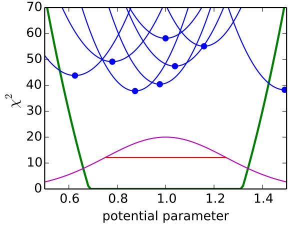


He argues that in order to obtain *model-independent* constraints on the potential, one needs to marginalize over *all* possible choices of DF instead of considering only the best-fit one.

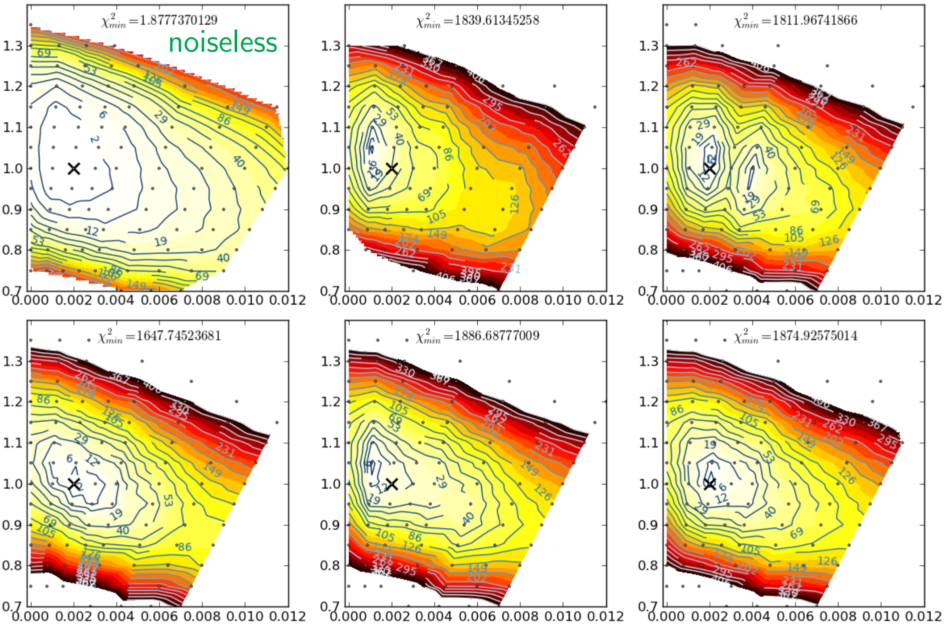
A proof-of-concept in the context of Made-to-measure models is given in Bovy, Hunt & Kawata 2018

A possibly more practical alternative is bootstrapping:

- repeat the fit for different subsets of data
- find the best-fit solution for each subset
- use the distribution of parameters of these best-fits



Measurement of gravitational potential: bootstrapping

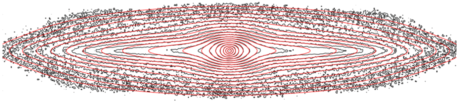


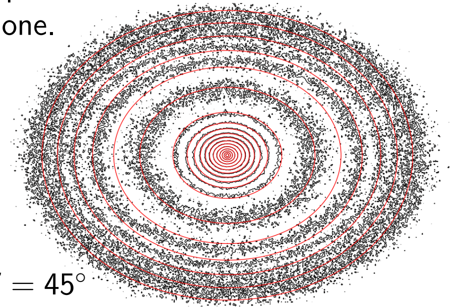
Uncertainties and biases in deprojection

Deprojection is **not unique** even in the axisymmetric (except edge-on) case!

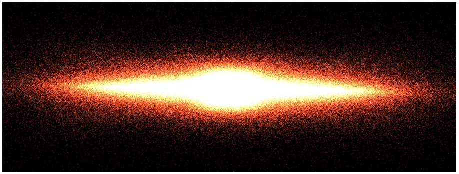
[Kochanek & Rybicki 1996; Gerhard & Binney 1996]

Multi-Gaussian expansion gives only one possible deprojection, but not necessarily a good one.

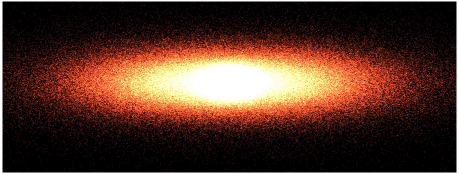




$i = 90^\circ$ $\quad$ $i = 45^\circ$



actual density profile

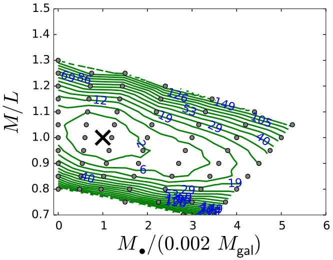


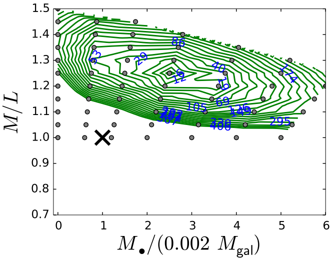
deprojected from MGE

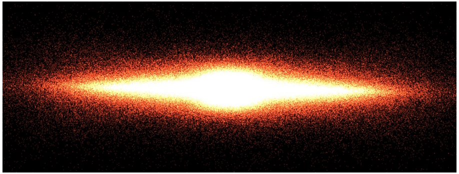
Measurement of potential

If the shape of the 3d mass distribution is “guessed” correctly, the potential parameters are recovered well, although with large uncertainty intervals.

If the shape is wrong, the potential is **tightly constrained but biased**.







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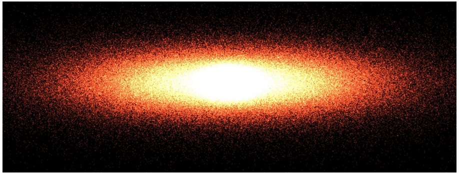


deprojected from MGE

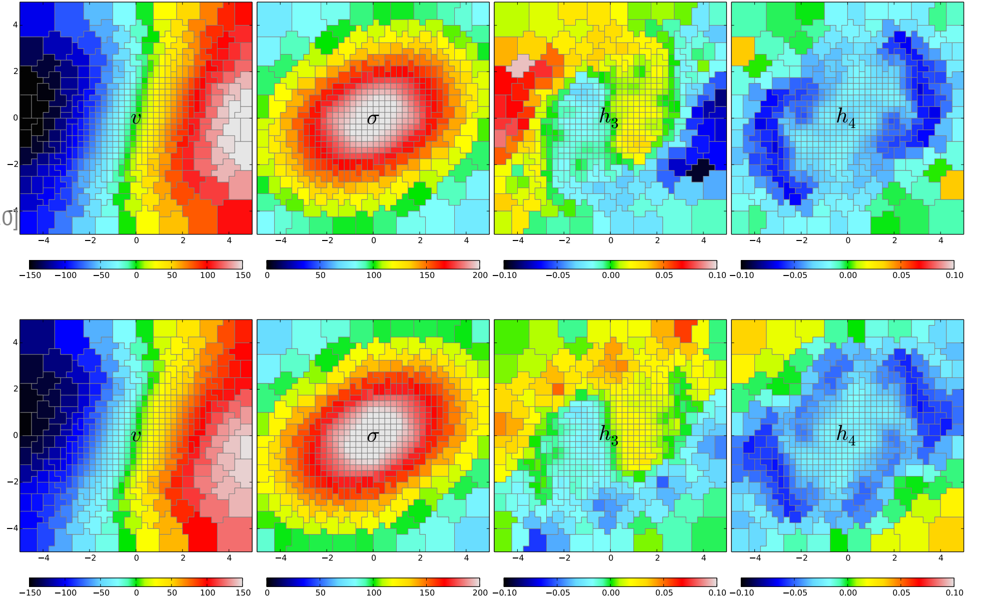
Measurement of pattern speed

inclination 45°, bar rotated by 45°

data

[Shen+ 2010]

model $\dfrac{\Omega}{\Omega_{\text{true}}} =$ 0.7



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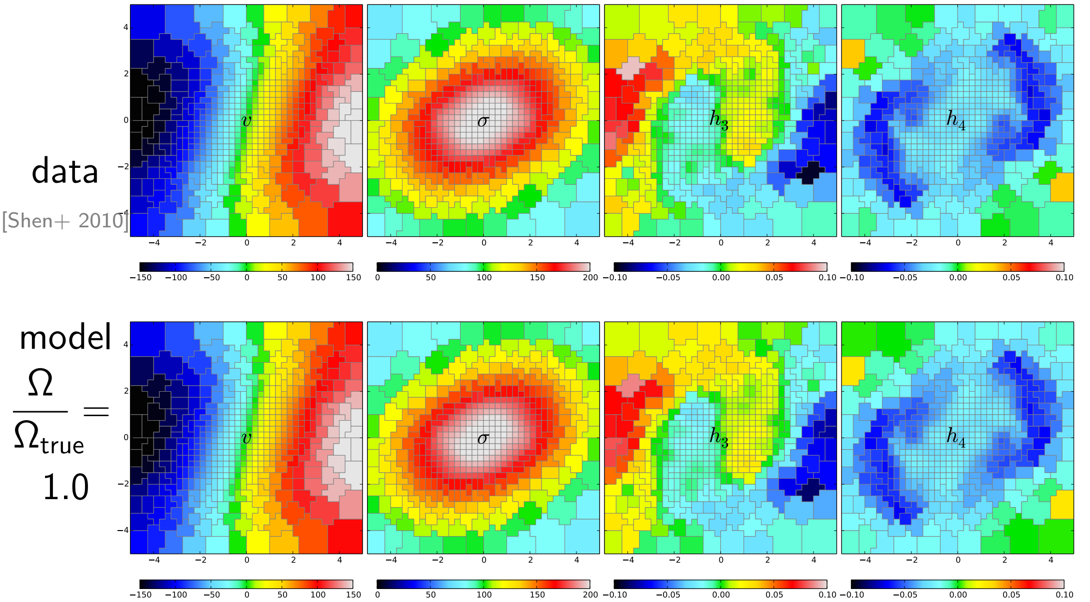

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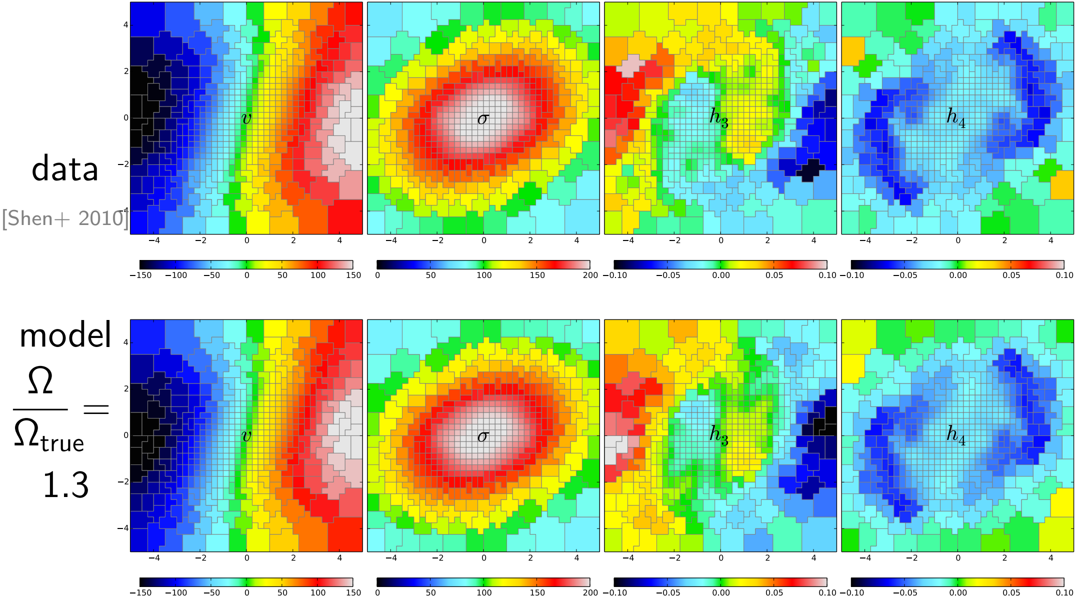

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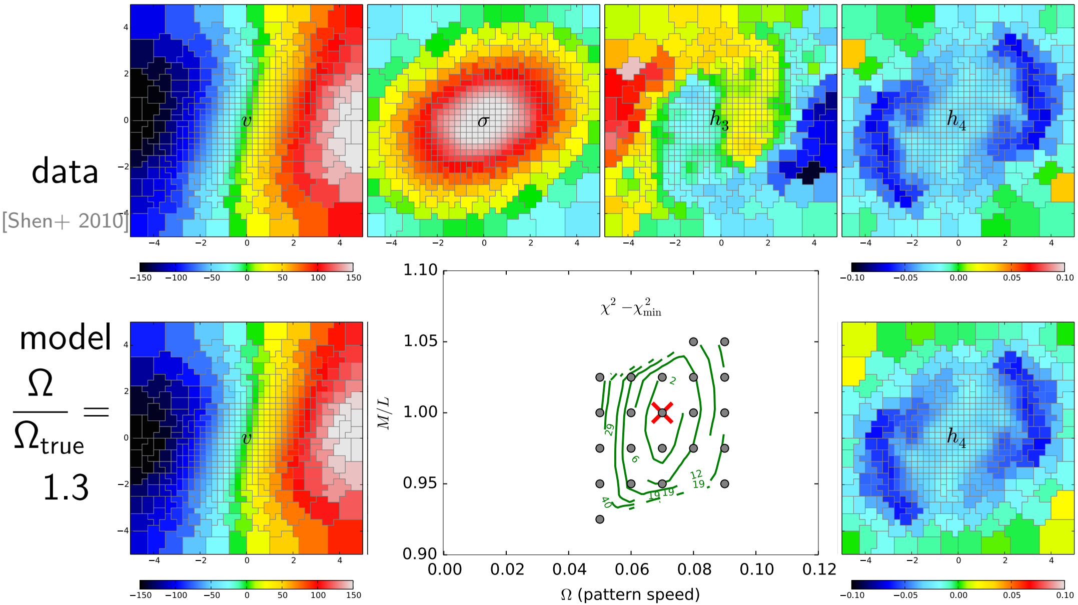

Summary

- Deprojection is the biggest uncertainty and source of systematic error
- Ellipsoidally-stratified density profiles are not appropriate for bars
- Pattern speed of a bar is relatively well constrained
- Large intrinsic degeneracy in SMBH mass measurement (in absense of noise)
- Bootstrapping may help to estimate uncertainties more realistically
- New implementation of Schwarzschild's method available to the community