Kinetics for gravity

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Gravity vs. plasma

$$\ddot{\mathbf{r}} = -\frac{G\, m_1\, m_2}{m_1\, |\mathbf{r}|^2}$$

single kind of charge

equal inertial and gravitating mass

no screening; Jeans length comparable to the size of the system

$$\ddot{\mathbf{r}} = \frac{q_1\, q_2}{m_1\, |\mathbf{r}|^2}$$

opposite charges

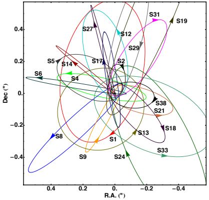
different charge/mass ratio

Debye screening length much smaller than the size of the system

In both cases $n\lambda^3 \gg 1 \;\Rightarrow$ nearly collisionless dynamics;

non-Maxwellian and possibly anisotropic velocity distributions.

Characteristic scales of self-gravitating systems



galactic nuclei

star clusters

galaxies

	star clusters	galaxies
	$N \sim 10^3 - 10^7$	$N \sim 10^6 - 10^{11}$
Size	$\sim 1 - 10\,\mathrm{pc}$	stars: $\sim 0.1 - 10$ kpc halo: $\sim 1 - 100$ kpc
Velocity	$\sim 10\,\mathrm{km/s}$	$\sim 10 - 200$ km/s
Dynamical time $\tau \equiv (4\pi\, G\, \rho)^{-1/2}$	$\sim 10^6$ yr	$\sim 10^8$ yr
Relaxation time $T_{\mathrm{rel}} \sim N\tau/\ln\Lambda$	$\sim 10^9 - 10^{10}$ yr	$\gg 10^{10}$ yr

Dynamical time (both): dynamically old

Relaxation time (both): thermodynamically evolving

$$\frac{1\,\mathrm{pc}}{1\,\mathrm{Myr}} \approx 0.98\,\mathrm{km/s}$$

Key features of kinetics of self-gravitating systems

Kinetic equation for $f(\mathbf{x}, \mathbf{v}, t)$:
$$\frac{\partial f}{\partial t} + \mathbf{v}\,\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}}\,\frac{\partial f}{\partial \mathbf{v}} = \mathfrak{C}_{\mathsf{coll}}[f].$$

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But: stars don't move on straight lines $\Rightarrow$
better use *orbits* J and *locations on the orbit* θ as phase-space variables.
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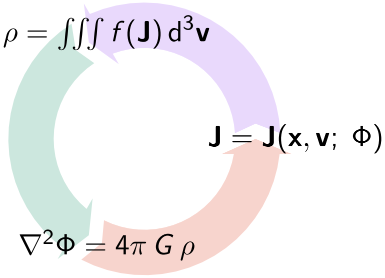
Solution of the CBE + Poisson equation:



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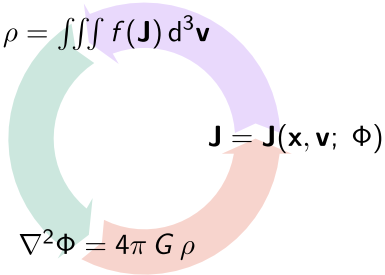
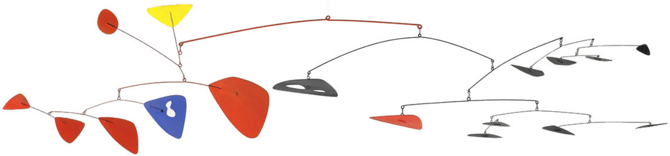
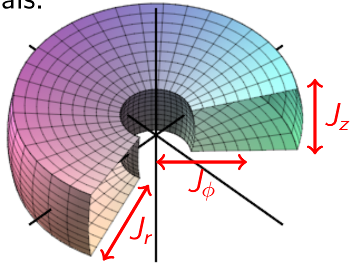


Alternative methods for constructing
equilibrium models: $\rho(\mathbf{x}) + \Phi(\mathbf{x}) \;\Rightarrow\; f(\mathbf{J})$



Action–angle variables in galactic dynamics

Typically we consider spherical or axisymmetric potentials.



- radial action J_r;
- vertical (or polar) action J_z;
- azimuthal action $J_\phi = L_z$
  (conserved component of angular momentum).

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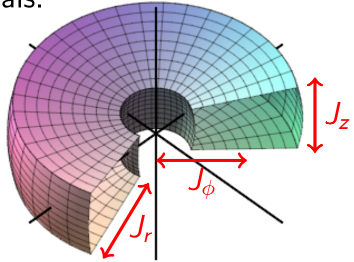

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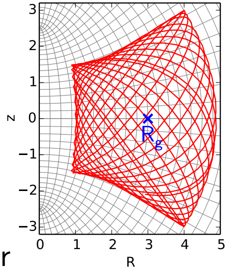
Transformation $\{\mathbf{x}, \mathbf{v}\} \Leftrightarrow \{\mathbf{J}, \boldsymbol{\theta}\}$:

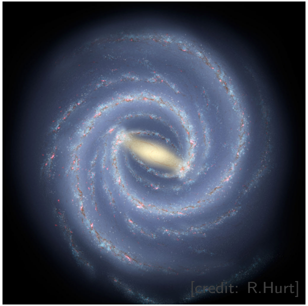


- for spherical potentials – (almost) analytic,
  only 1d numerical integrals for J_r;
- for axisymmetric potentials close to the equatorial plane – epicyclic approximation (separable motion in R and z);
- —"—, Stäckel approximation (spheroidal coordinate system);
- most general (but tricky to use): "torus mapping" using Fourier series for generating functions of the canonical transformation.

Observational developments in recent years

gaia + many ground-based spectroscopic surveys

proper motions	distances	line-of-sight velocities	ages, chemical composition
$\mathcal{O}(10^9)$	$\mathcal{O}(10^8)$	$\mathcal{O}(10^7)$	$\mathcal{O}(10^6)$



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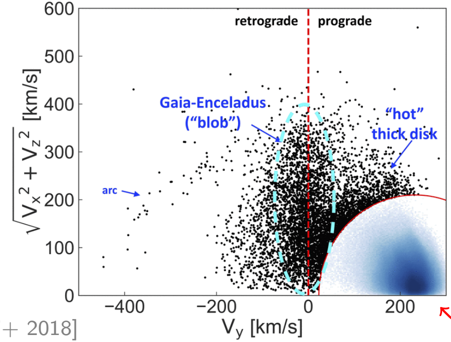
Available data: significant part of the Galactic disc ($\sim$ few kpc); central region, outer halo, some satellites...

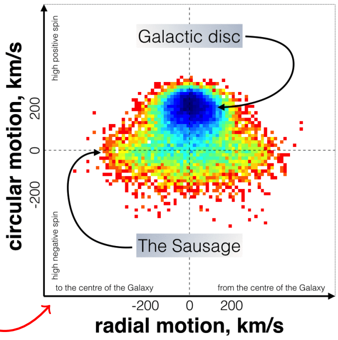
Challenges: patchy coverage; not all objects have 6d phase-space coords...

Recent discoveries: Galactic assembly history



[Helmi+ 2018]

turn 90°



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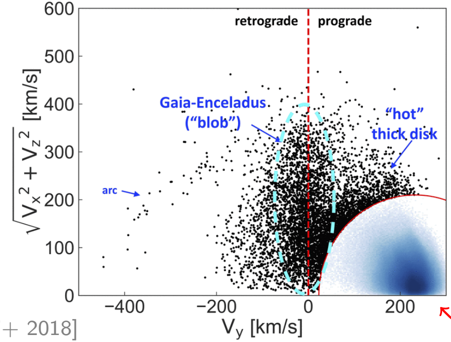

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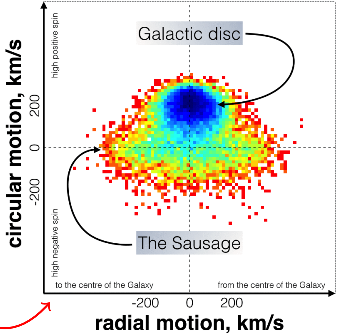

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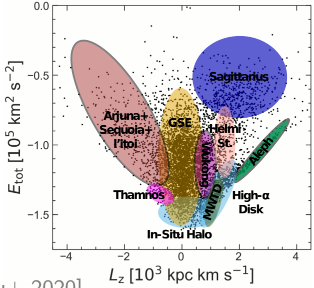
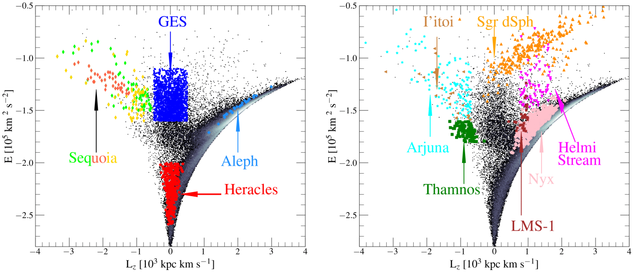


[Naidu+ 2020]



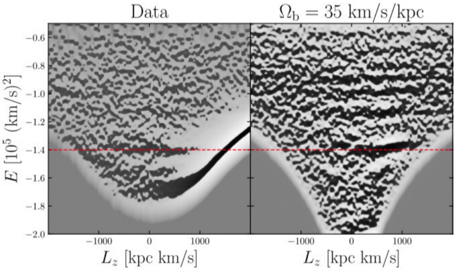
[Horta+ 2023]

and many others...

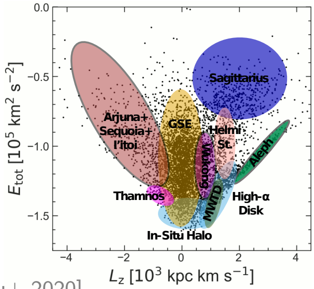
Recent discoveries: Galactic assembly history

Challenges:

– overlapping debris from multiple progenitors;
– incomplete phase mixing;
– limited spatial coverage;
– blurring of substructures by later perturbations;
– dynamical creation of structures;
– lack of consistent nomenclature...



bar-induced stripes [Dillamore+ 2023, 2024]



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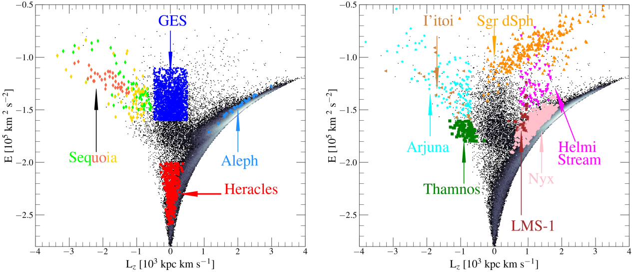

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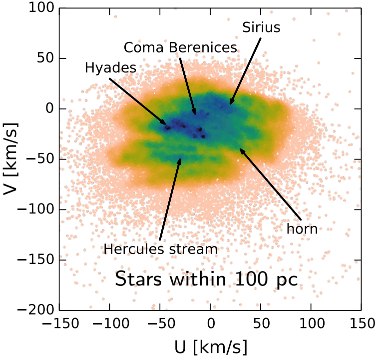
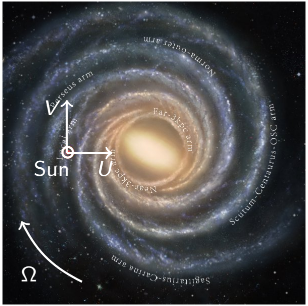
Recent discoveries: imprint of Galactic bar and spiral arms

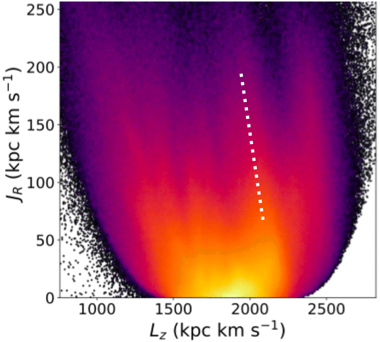
Velocity distribution in the equatorial plane (U, V) contains various structures associated with resonantly trapped orbits [Dehnen 2000; Quillen & Minchev 2005].

In a more extended region, trapped orbits show up as lines in the $J_r - L_z$ plane [e.g., Sellwood 2010; Binney 2018; Monari+ 2017, 2019; Trick+ 2019; Hunt+ 2019].

Their location depends on the pattern speed of the bar Ω_b and spiral arms Ω_s.
A slowing-down bar transports trapped objects outwards [e.g., Dillamore+ 2024]
and creates age-dependent structures in the resonant islands [e.g., Chiba+ 2021].







[adapted from Trick+ 2019]

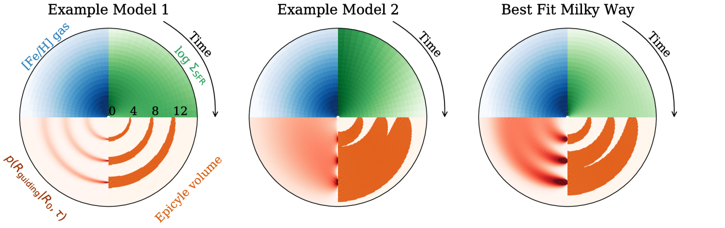
Recent discoveries: radial migration and disc heating

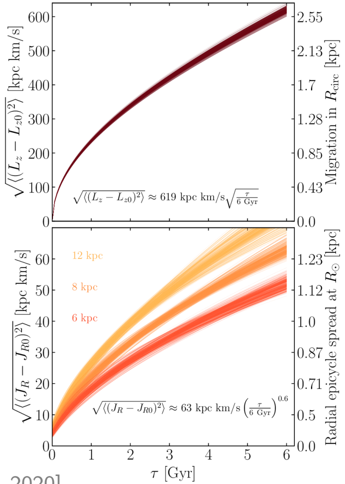
Stars do not stay at the same near-circular orbits where they were born due to a combination of two effects [Sellwood & Binney 2002; Roškar+ 2008; Minchev & Famaey 2010]:

– radial migration ("churning" – change in L_z while conserving J_r);
– heating ("blurring" – diffusion in J_r).

Mechanisms:
– resonances with the bar and spiral arms;
– molecular clouds and other massive perturbers;
– external perturbations (e.g., satellite flybys).

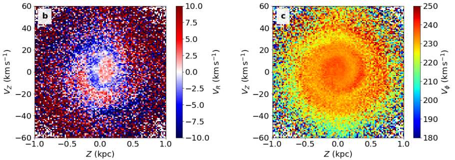
Churning appears to be much stronger than blurring.



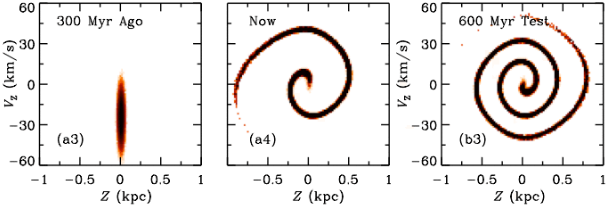


[Frankel+ 2020]

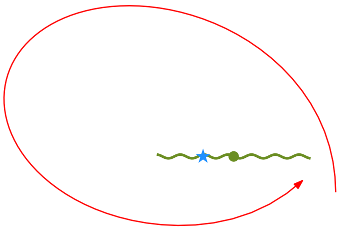
Recent discoveries: vertical perturbations in the Galactic disc



Gaia DR2 [Antoja+ 2018]



[Li & Shen 2019]



Leading theory: ripples after the impact of a massive satellite (implying Sgr dSph) through the disc [Widrow+ 2012; Laporte+ 2018,2019; Binney & Schönrich 2018; Li & Shen 2019; Bland-Hawthorn & Tepper-García 2021, etc.]

Caveat: Sgr was likely not massive enough at the time of the previous passage through the disc (1 Gyr ago) [Vasiliev & Belokurov 2020; Bennett+ 2022].

Counter-caveat: Sgr may have excited long-lived oscillations in the MW halo, which in turn perturb the disc [Grand+ 2022].

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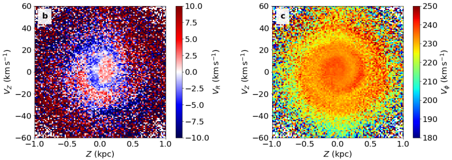

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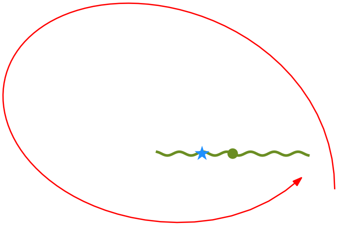
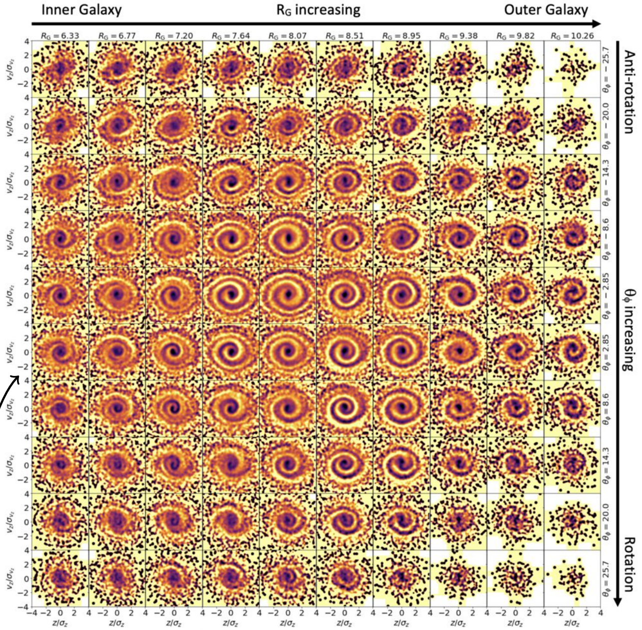




two-arm spiral in the inner Galaxy:
breathing perturbation due to the bar?

Gaia DR3 [Hunt+ 2022]

Recent discoveries: precessing warp in the outer Galactic disc

The warp is a coherent large-scale vertical perturbation of the disc beyond $\sim 10-12$ kpc.

Formation theories:

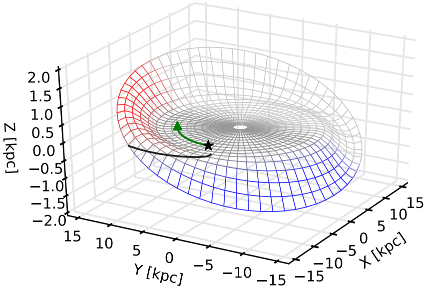
– impact of satellites (Sgr, LMC);

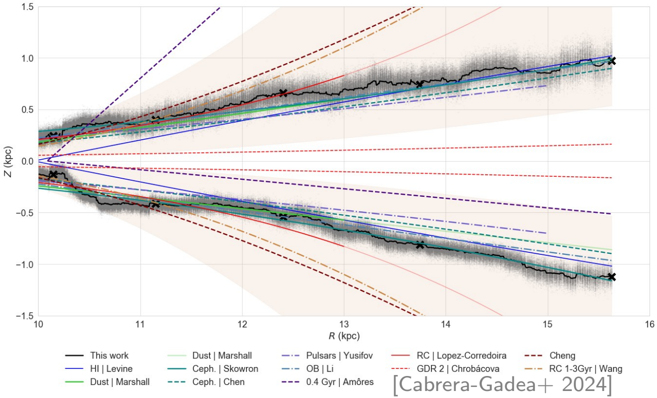
– misalignment between the disc and the dark halo;

– cold gas accretion from a misaligned direction.

Challenges:

– disagreement in amplitude and precession rate between populations of different ages.



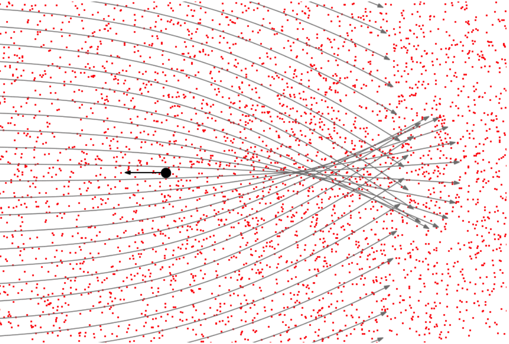


[Cabrera-Gadea+ 2024]

Recent discoveries: LMC–Milky Way encounter

The Large Magellanic Cloud is only $5 - 10\times$ less massive than the Milky Way, and just passed its pericentre at 50 kpc. The LMC-induced perturbation is twofold:

1. Stars in the vicinity of the moving LMC are deflected into a trailing density wake, creating a dynamical friction force [Chandrasekhar 1942].



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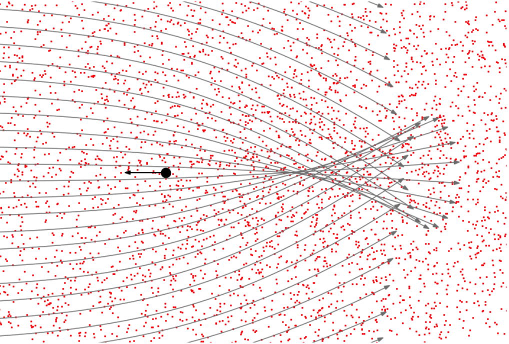
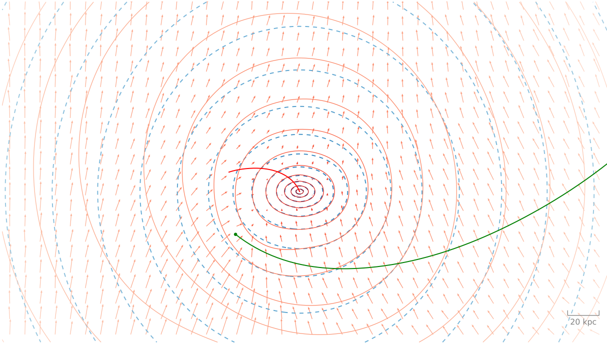
In the MW-centred reference frame, outer halo appears to move up (dipole perturbation).





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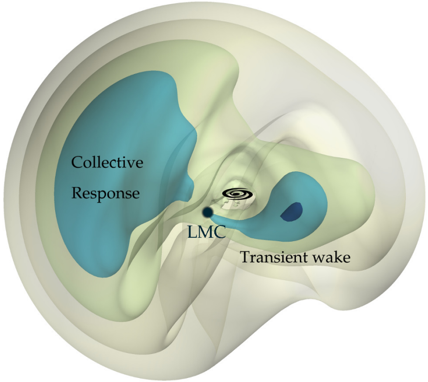
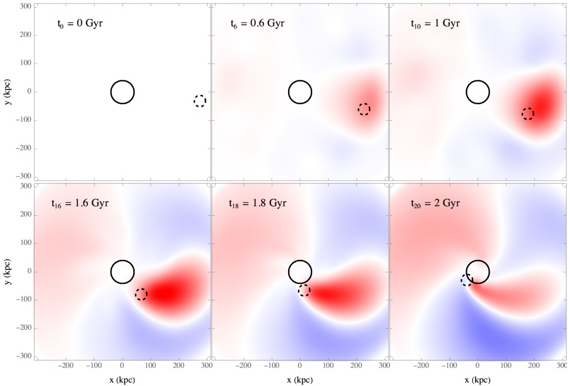


N-body sims [Garavito-Camargo+ 2020]



perturbation theory [Rozier+ 2022]

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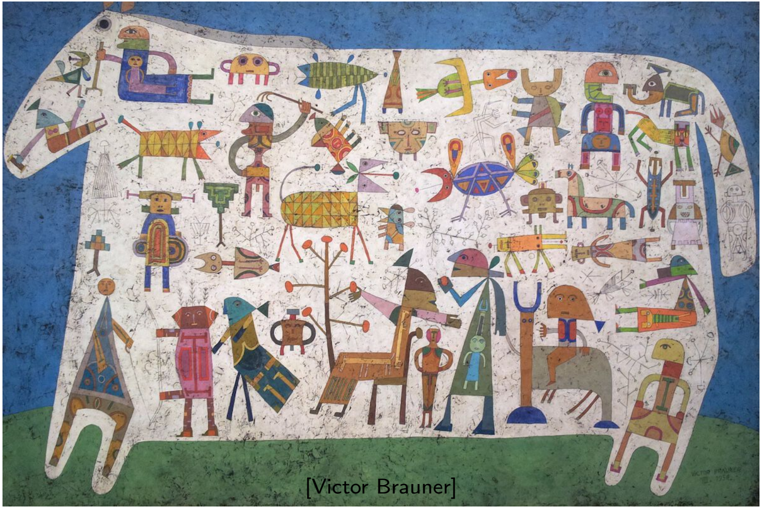

[Victor Brauner]