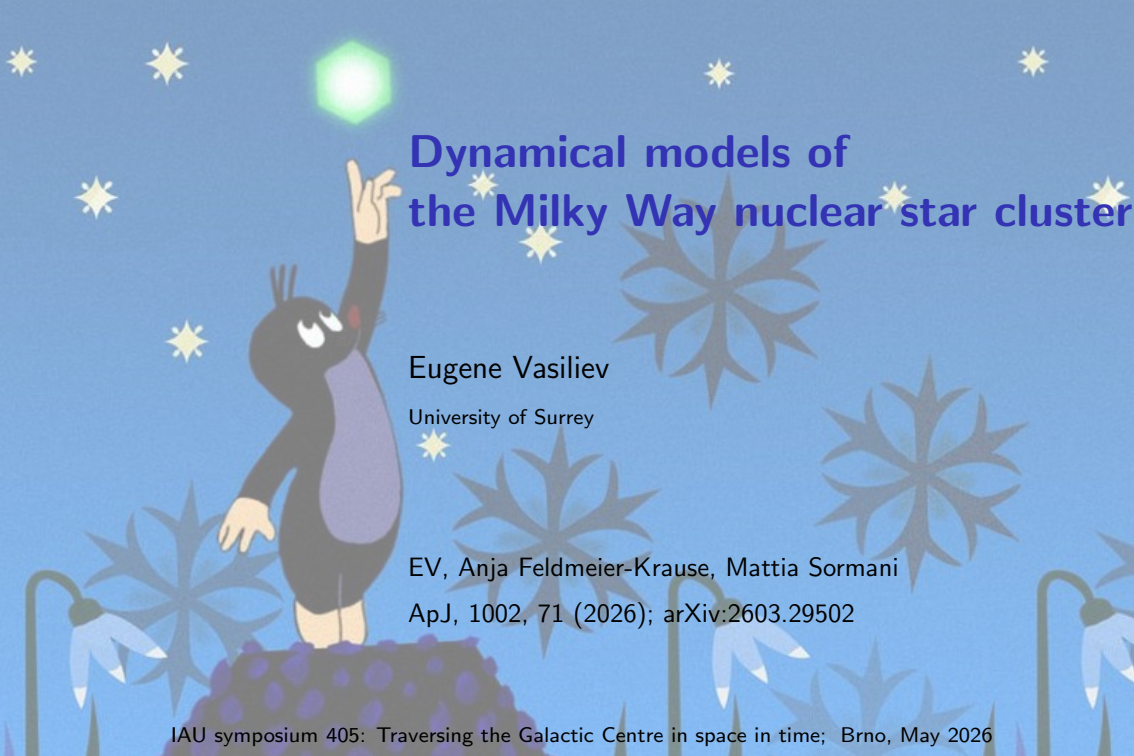


Dynamical models of the Milky Way nuclear star cluster

Eugene Vasiliev

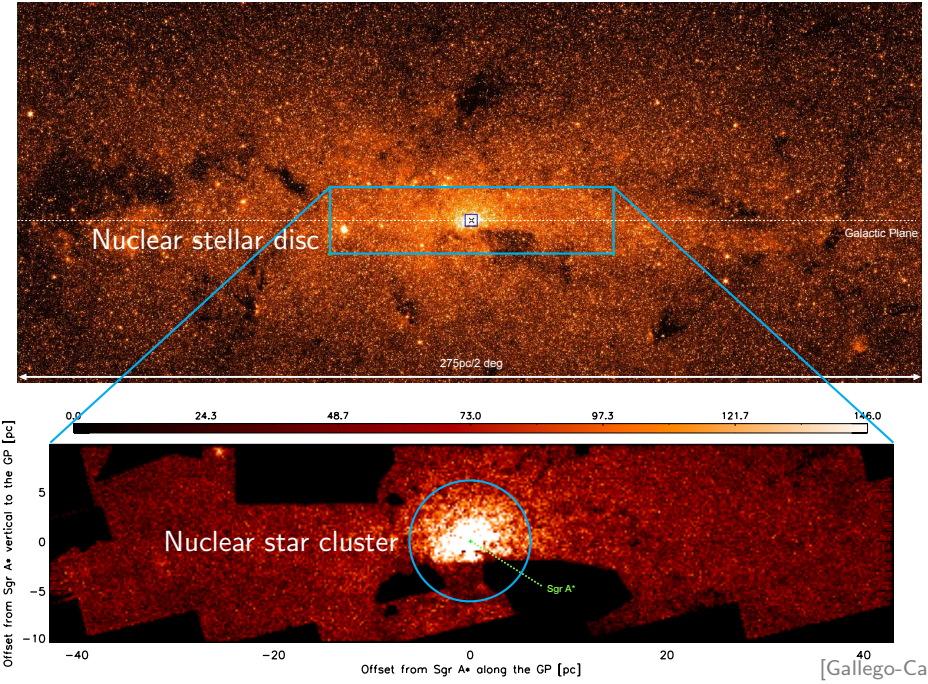
University of Surrey

EV, Anja Feldmeier-Krause, Mattia Sormani

ApJ, 1002, 71 (2026); arXiv:2603.29502

IAU symposium 405: Traversing the Galactic Centre in space in time; Brno, May 2026

Nuclear star cluster of the Milky Way



[Gallego-Cano+ 2020]

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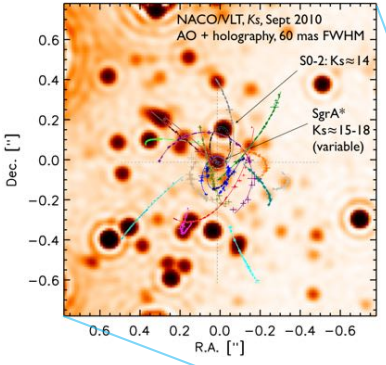
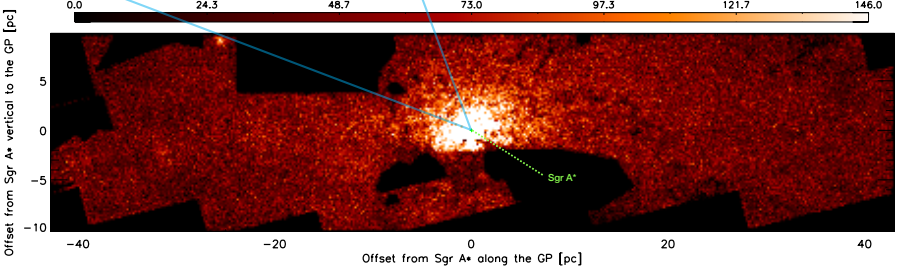


orbits of
$\mathcal{O}(10)$ S-stars
[Genzel+; Ghez+;
GRAVITY collab.]



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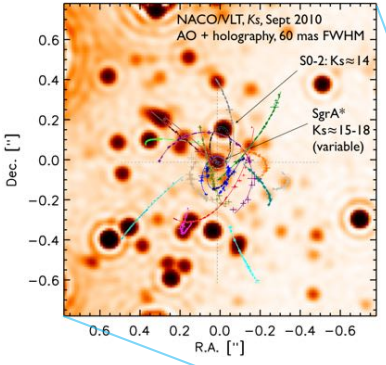

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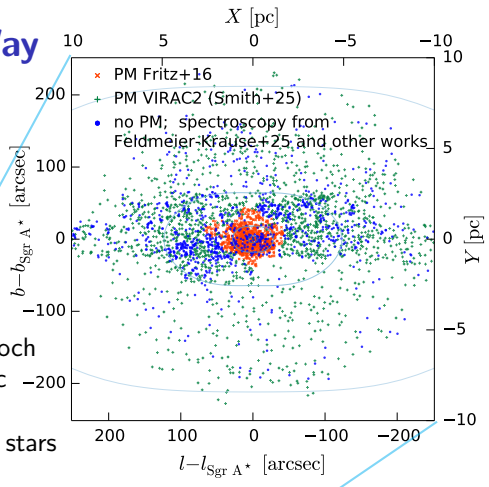
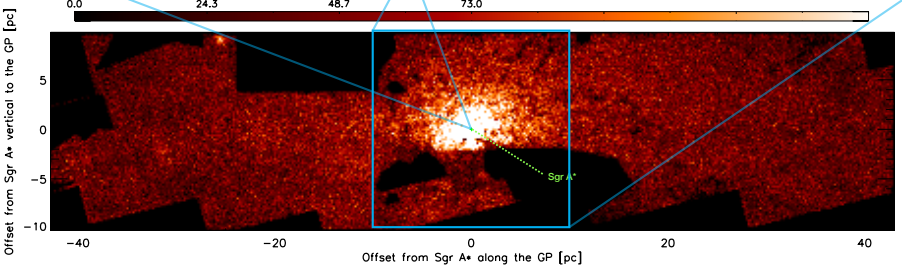
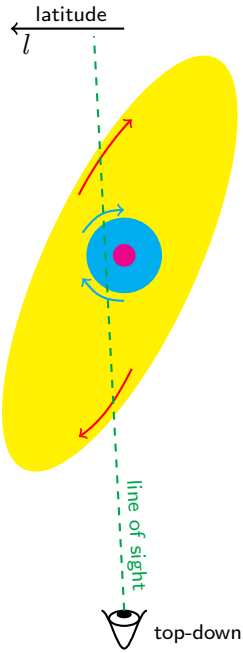
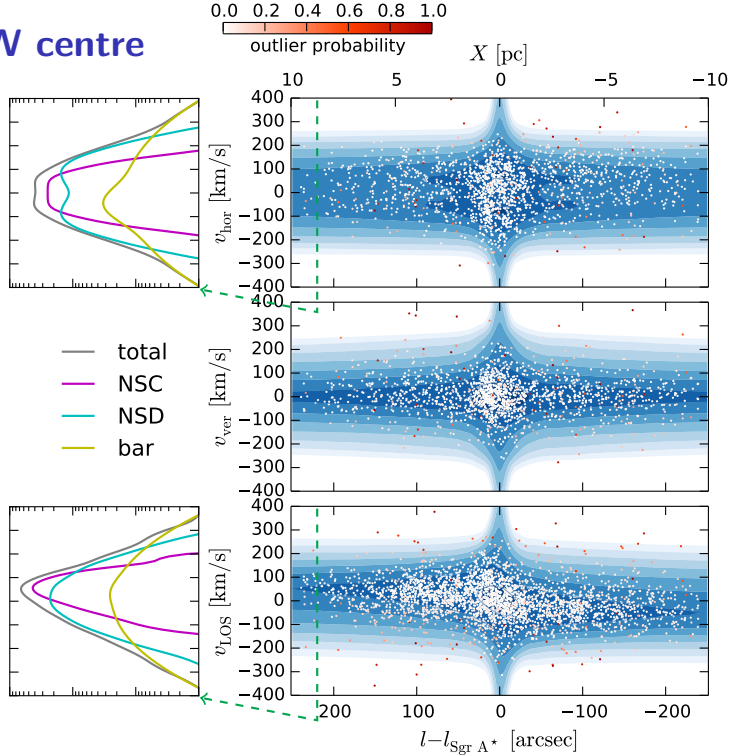


single-epoch
kinematic
snapshot
few$\times 10^3$ stars



Kinematics in the MW centre





Dynamics in the MW centre

Goal: constrain the mass distribution (M_{BH}, stars, stellar remnants) and the orbital structure of the NSC.

Methods:

- orbit fitting of S-stars in the vicinity of Sgr A*
  [Genzel+ 2003+; Ghez+ 2003+; GRAVITY 2018+]
- Jeans models of the NSC [Schödel+ 2009; Do+ 2013, 2020; Feldmeier+ 2014, 2025; Fritz+ 2016] and NSD [Sormani+ 2020]
- Schwarzschild orbit-superposition models of the NSC [Feldmeier-Krause+ 2017] and NSD [Feldmeier-Krause+ 2026]
- Distribution function-based models of the NSC [Chatzopoulos+ 2015; Magorrian+ 2019; Vasiliev+ 2026] and NSD [Sormani+ 2022]

Distribution functions-based self-consistent models

DF $f(\mathbf{x}, \mathbf{v})$ offers a complete description of the stellar population:

- density $\rho(\mathbf{x}) = \iiint f(\mathbf{x}, \mathbf{v})\, \mathrm{d}^3 v$,
- velocity moments $\left\{\overline{\mathbf{v}},\ \overline{v_i\, v_k}\right\}(\mathbf{x}) = \dfrac{1}{\rho(\mathbf{x})} \iiint \left\{\mathbf{v},\ v_i\, v_j\right\} f(\mathbf{x}, \mathbf{v})\, \mathrm{d}^3 v$,
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Jeans's theorem: in an equilibrium state, DF $f(\mathcal{I})$ must be a function of integrals of motion $\mathcal{I}(\mathbf{x}, \mathbf{v};\ \Phi)$, e.g., energy, angular momentum, or actions.

Potential Φ is linked to density via the Poisson equation $\nabla^2 \Phi = 4\pi\, G\, \rho$.

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$$\left.\begin{array}{r} f_{\mathrm{NSC}}(\mathcal{I};\ \boldsymbol{\alpha}) \\ f_{\mathrm{NSD}}(\mathcal{I};\ \boldsymbol{\beta}) \\ M_{\mathrm{BH}} \end{array}\right\}$$

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$$\Longrightarrow \begin{array}{l} \rho(\mathbf{x}) \\ \Phi(\mathbf{x}) \\ \mathfrak{f}(\mathbf{v}\,|\mathbf{x}) \end{array}$$

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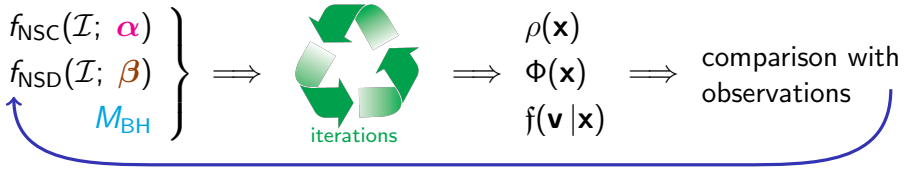


adjust NSC parameters $\boldsymbol{\alpha}$ and BH mass in a MCMC loop

NSD parameters $\boldsymbol{\beta}$ are fixed (determined by Sormani+ 2022)

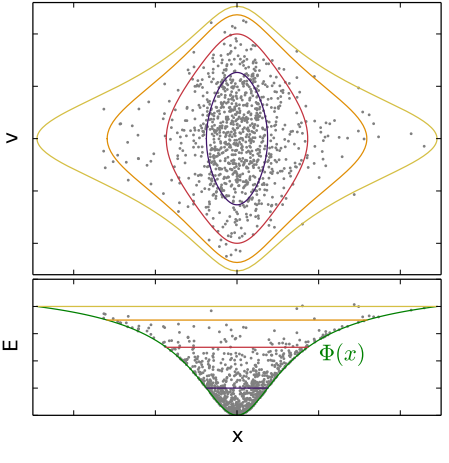
Likelihood of a DF-based model

DF is a probability distribution $\Longrightarrow$

$$\ln \mathcal{L} = \sum_{i=1}^{N_{\rm stars}} \ln f(\mathbf{x}_i, \mathbf{v}_i;\ \boldsymbol{\alpha}).$$

Maximise the likelihood of the entire dataset by varying DF parameters $\boldsymbol{\alpha}$.

Potential is constrained because DF depends on it through $\mathcal{I}$: $\quad f\big(\mathcal{I}(\mathbf{x}_i, \mathbf{v}_i;\ \Phi)\big)$.



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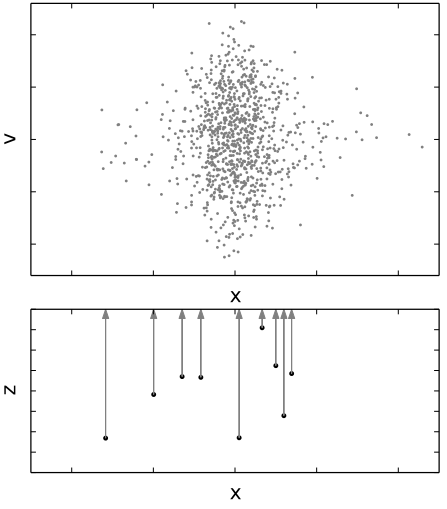
Complication #1: projection

$$f(x, v_z) = \int \mathrm{d}z \int \mathrm{d}v_x\ f(x, z, v_x, v_z).$$



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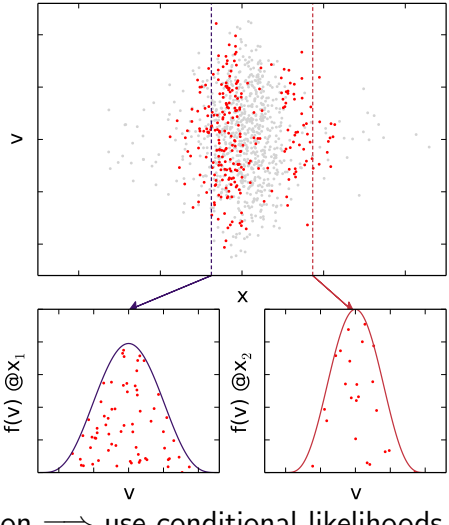

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Complication #2: [unknown] spatial selection $\Longrightarrow$ use conditional likelihoods

$$\mathcal{L}_i = \mathcal{P}(\mathbf{v}_i \mid \mathbf{x}_i) = \frac{f(\mathbf{x}_i, \mathbf{v}_i)}{\iiint \mathrm{d}^3\mathbf{v}\ f(\mathbf{x}_i, \mathbf{v})} = \frac{f(\mathbf{x}_i, \mathbf{v}_i)}{\rho(\mathbf{x}_i)}.$$

Potential constrained only by kinematic data, without knowing the density profile.

Results: NSC kinematics

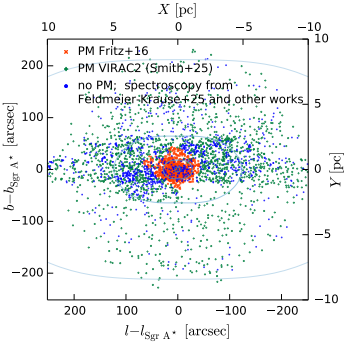
Two series of models:

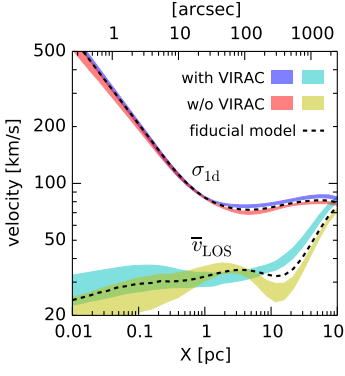
$\mathbb{F}$: $V_{\rm los}$ + PM from Fritz+ 2016 within $\sim$2 pc;

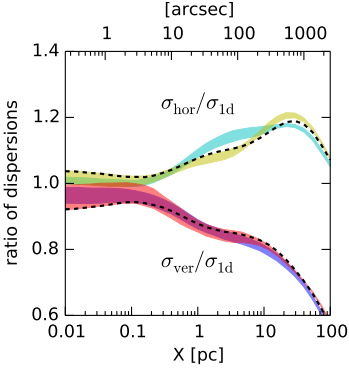
$\mathbb{V}$: also adding PM from VIRAC2 outside 2 pc.

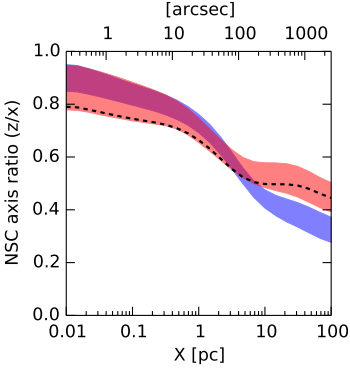
Rotation and velocity dispersions are similar in both.

PM anisotropy induces vertical flattening $q \simeq 0.8$





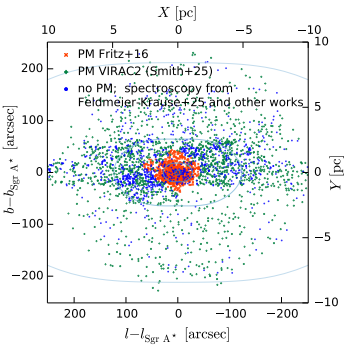


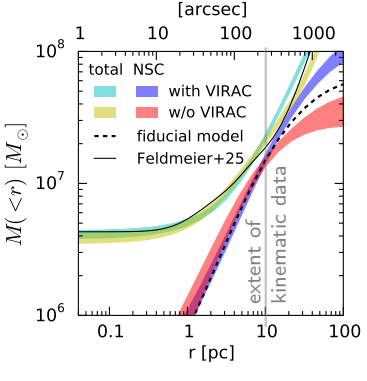


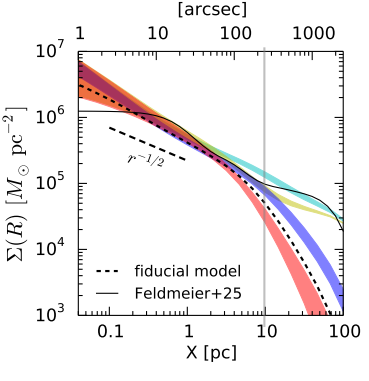
Results: mass distribution

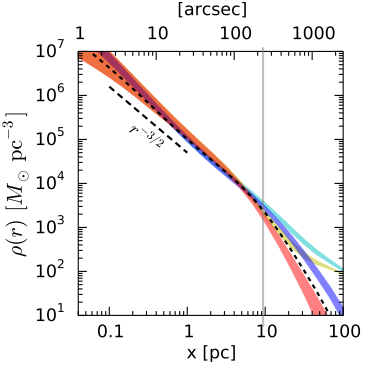
Two series of models:
$\mathbb{F}$: $V_{\rm los}$ + PM from Fritz+ 2016 within $\sim$2 pc;
$\mathbb{V}$: also adding PM from VIRAC2 outside 2 pc.

Both mass profiles are compatible within 10 pc,
but $\mathbb{V}$ favours higher total mass of NSC.
Inferred density $\rho(r) \propto r^{-3/2}$ matches observations.

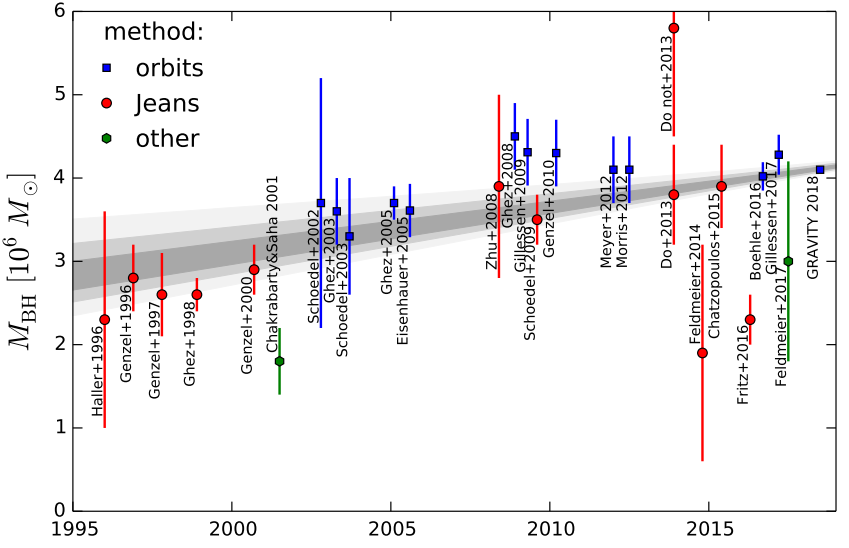








Black hole mass growth



(figure originally presented at the "Dense stellar systems" workshop in Heidelberg, November 2018)

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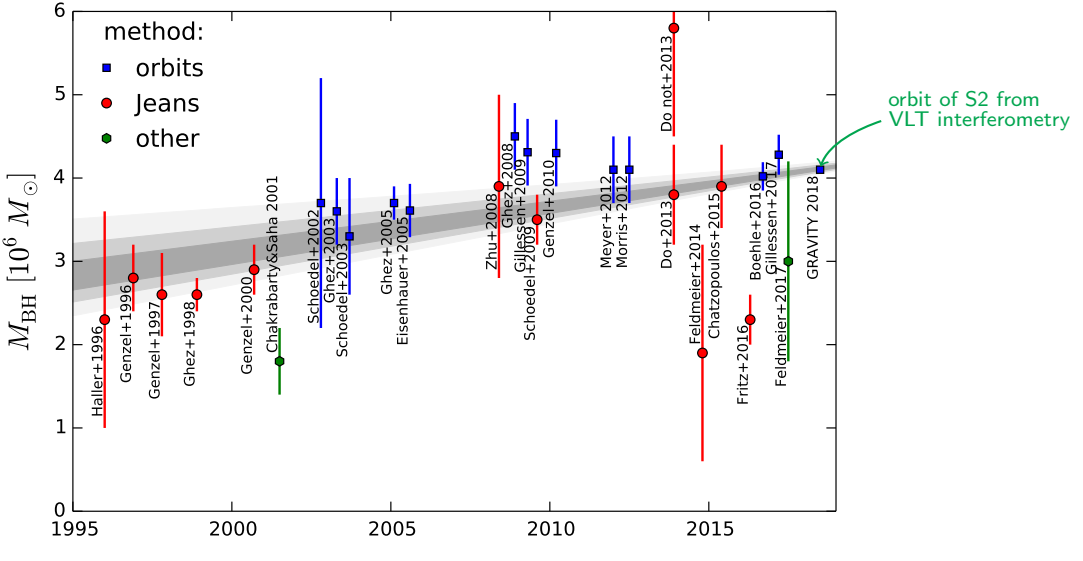

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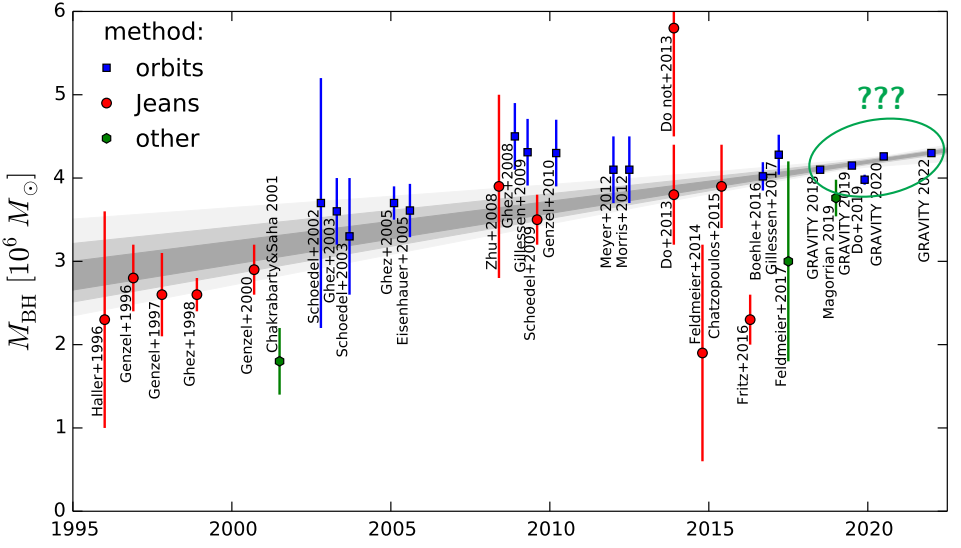
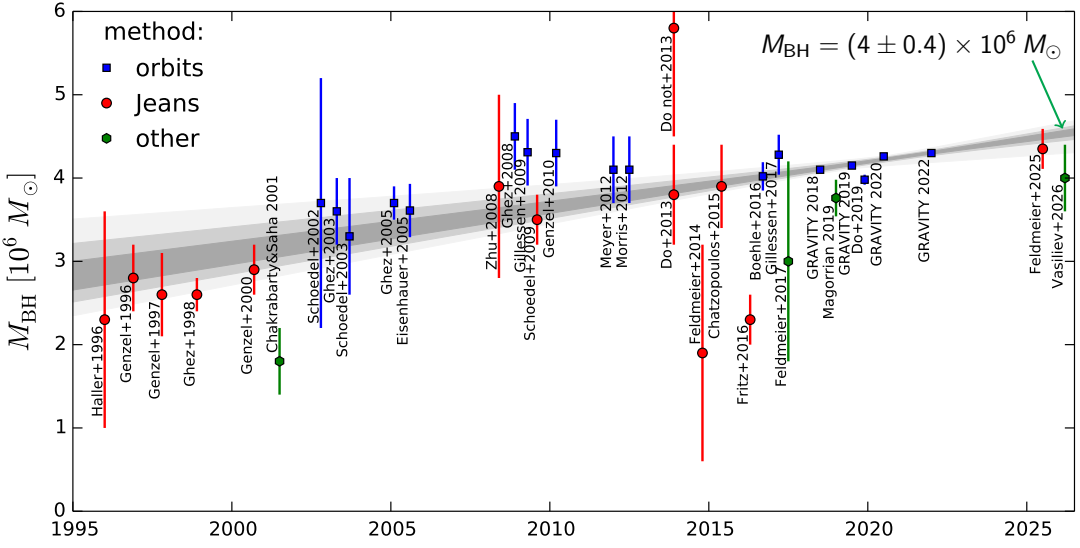


Black hole mass growth is curtailed!



For a longer story, see arXiv:2603.29503: "Milky Way evolution on a human timescale"

Summary

- First three-integral DF-based self-consistent model of the MW NSC.
- Dynamical model constrained only by discrete-kinematic data,
  but recovers the density profile well.
- Mass distribution is compatible with other works.
- Best-fit BH mass within 10% of the true value
  (but only after accounting for bar contamination! – cautionary tale for extragalactic $M_{\rm BH}$ measurements).
- Model available at
  `https://github.com/GalacticDynamics-Oxford/Agama/blob/master/py/example_mw_nsc.py`

[Kupka – Discs of Newton]