The slide features a decorative border on the left and right sides, composed of vertical strips of various patterns including wood grain, spirals, and geometric shapes. The background is a warm, golden-brown color with a subtle, out-of-focus pattern of stars and galaxies.

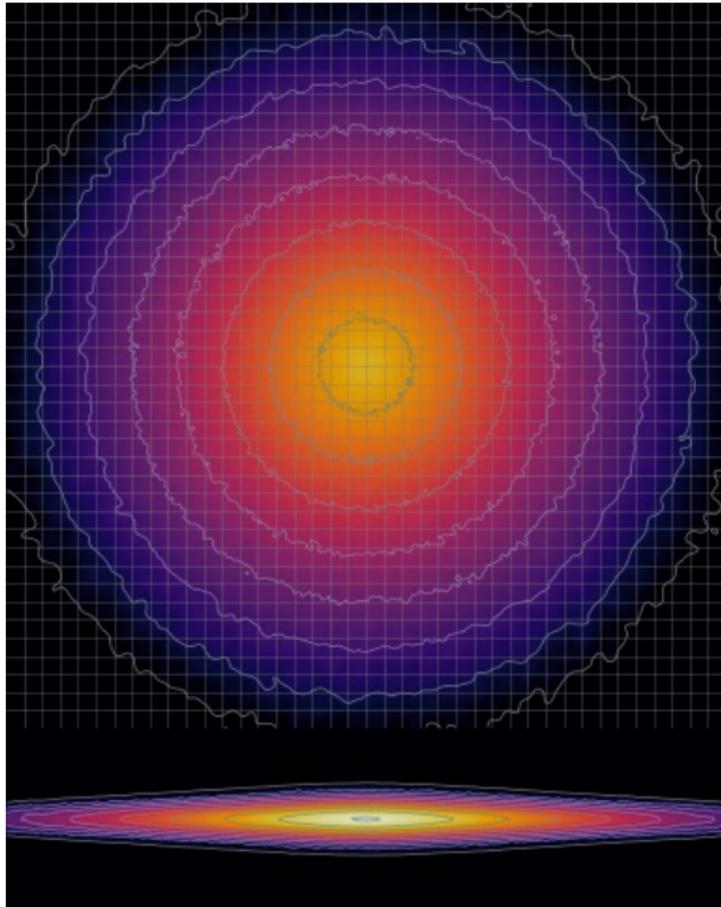
# Barred galaxies and how to model them

Eugene Vasiliev

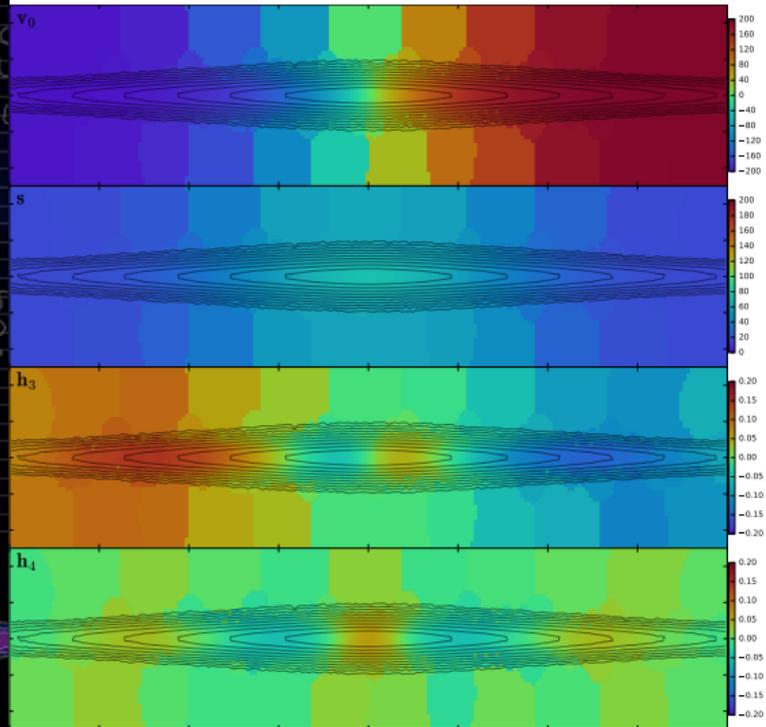
Institute of Astronomy, Cambridge

Vienna Dynamics Workshop, 8 October 2019

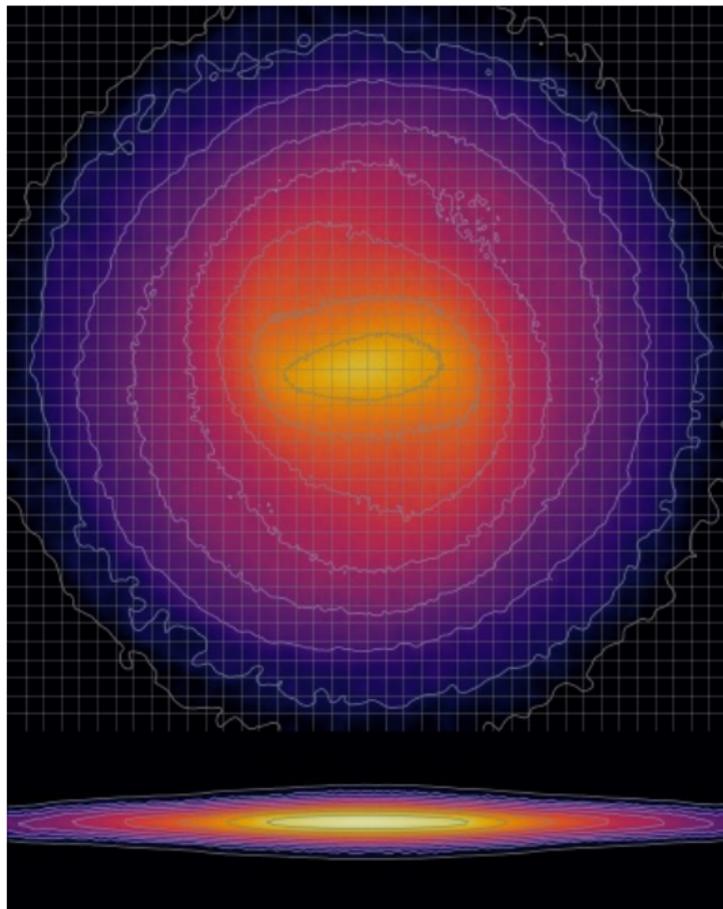
# The turbulent history of a bar-unstable galaxy model



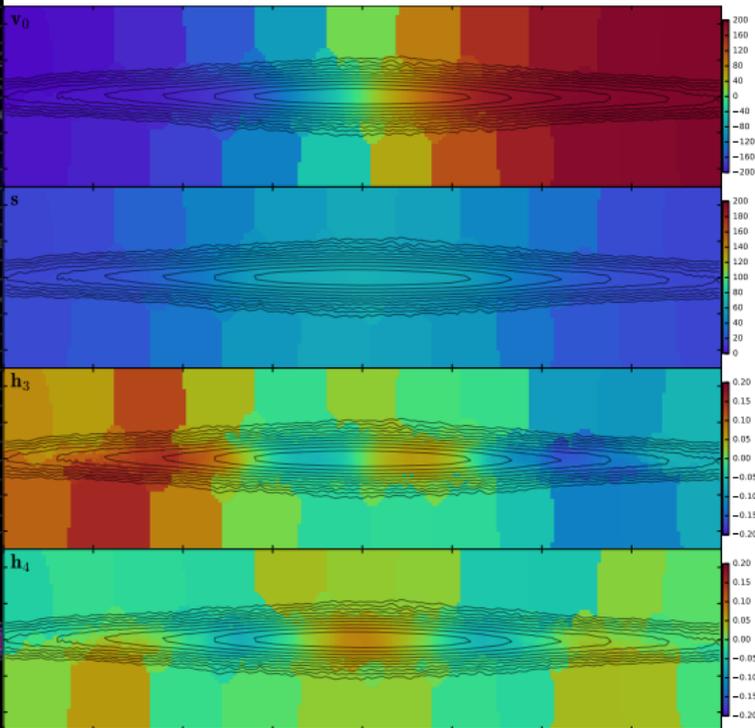
initially axisymmetric disk



# The turbulent history of a bar-unstable galaxy model

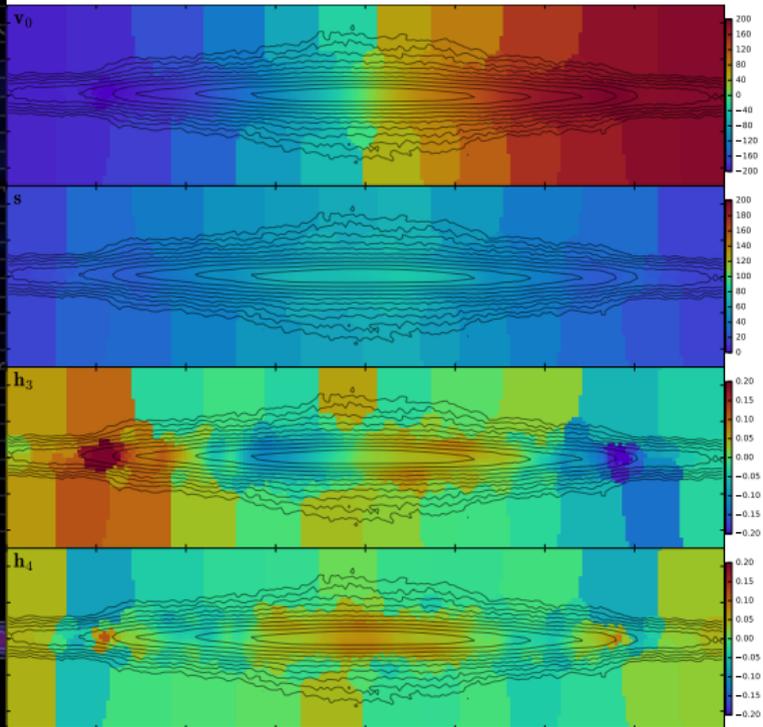
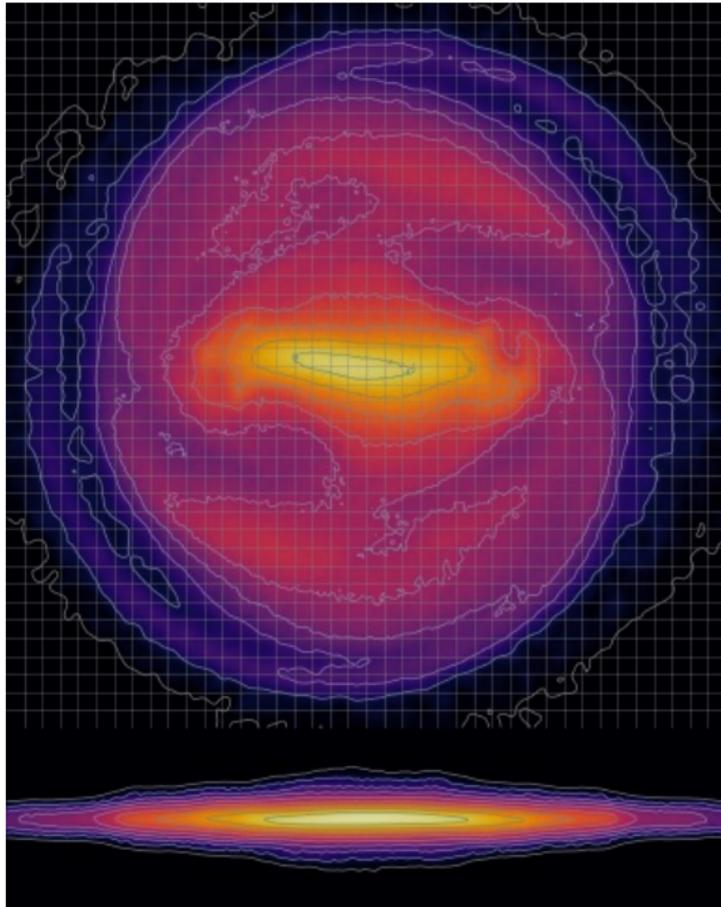


bar starts to develop



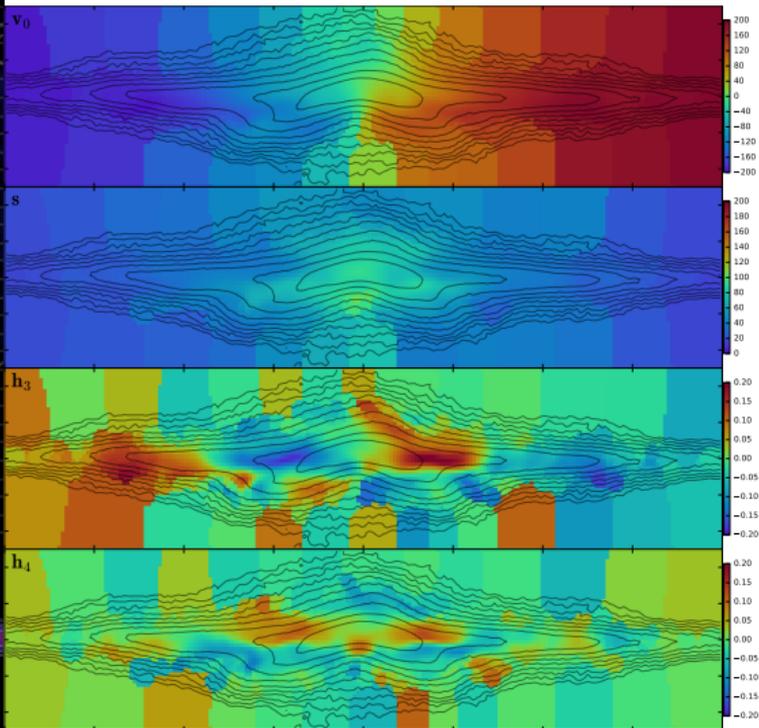
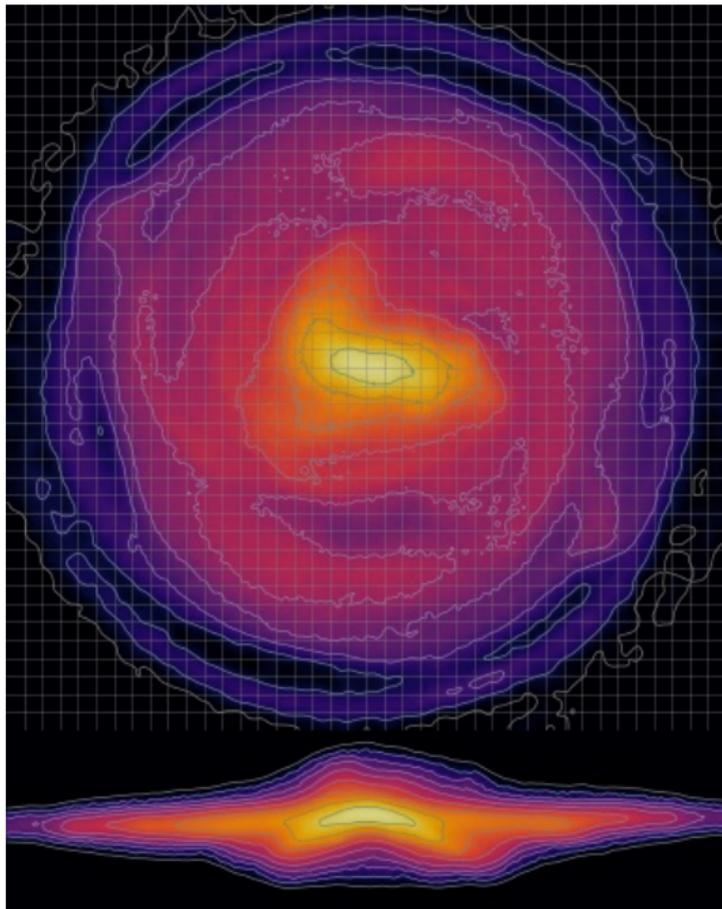
# The turbulent history of a bar-unstable galaxy model

bar grows long and thin



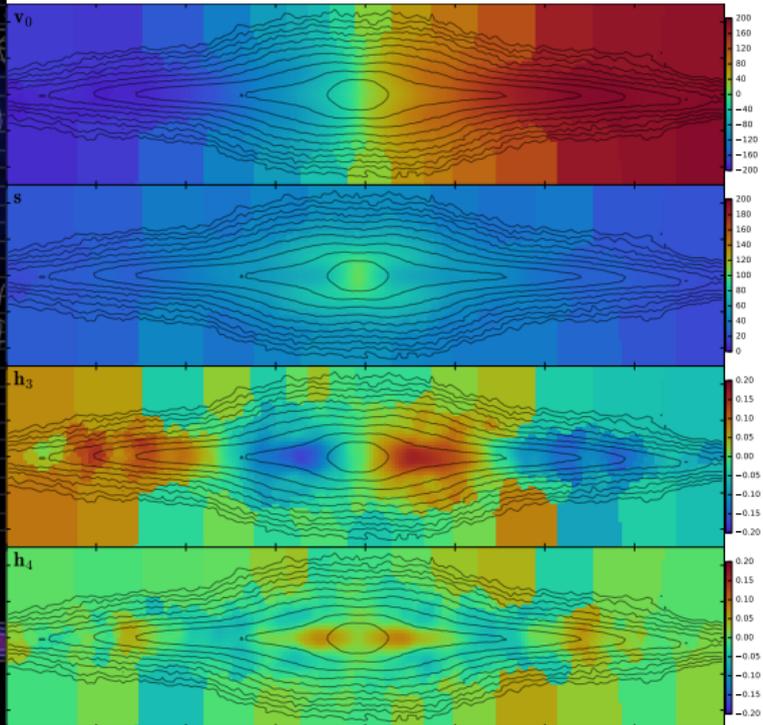
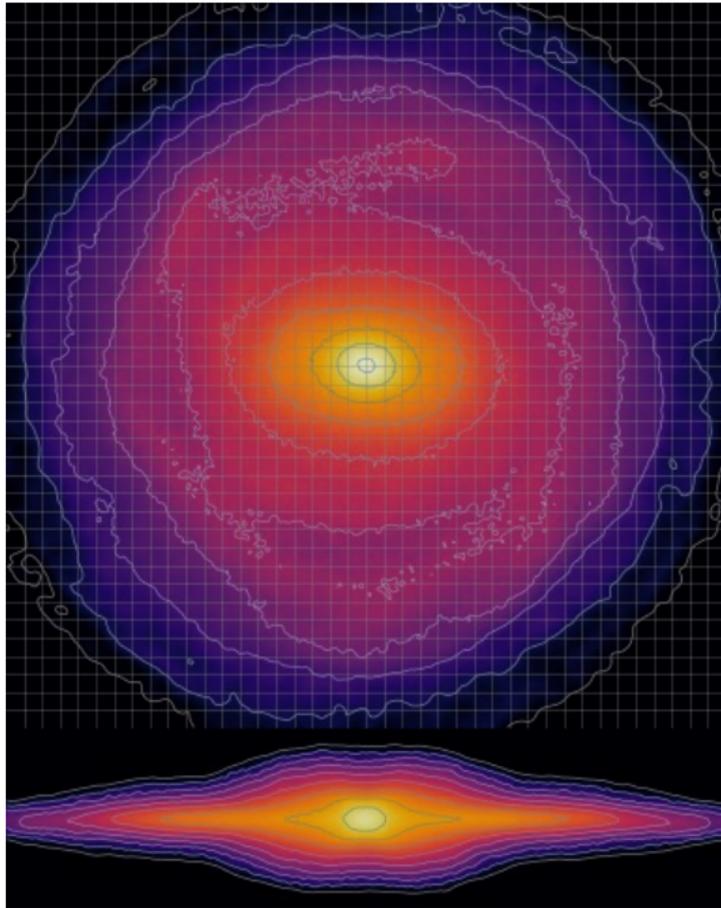
# The turbulent history of a bar-unstable galaxy model

buckles in the vertical direction



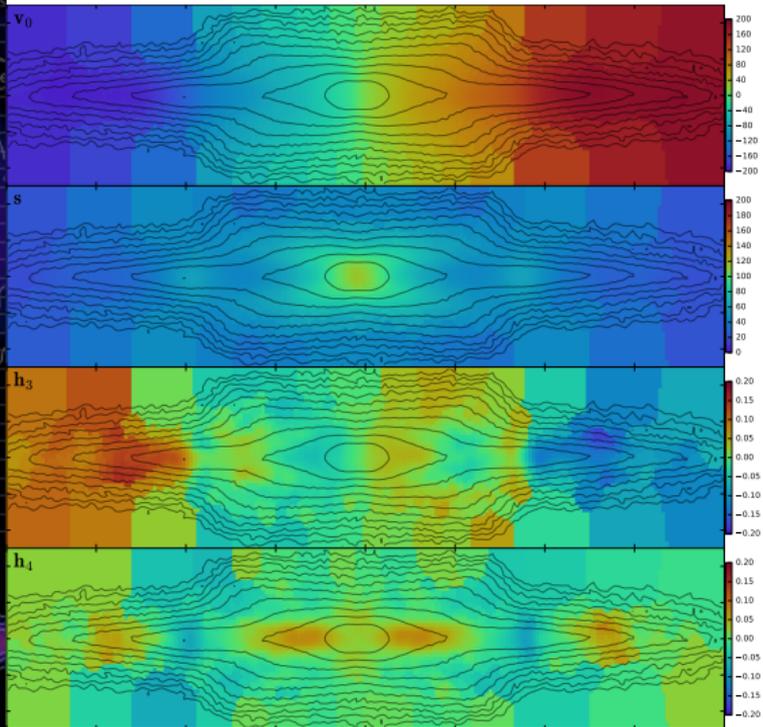
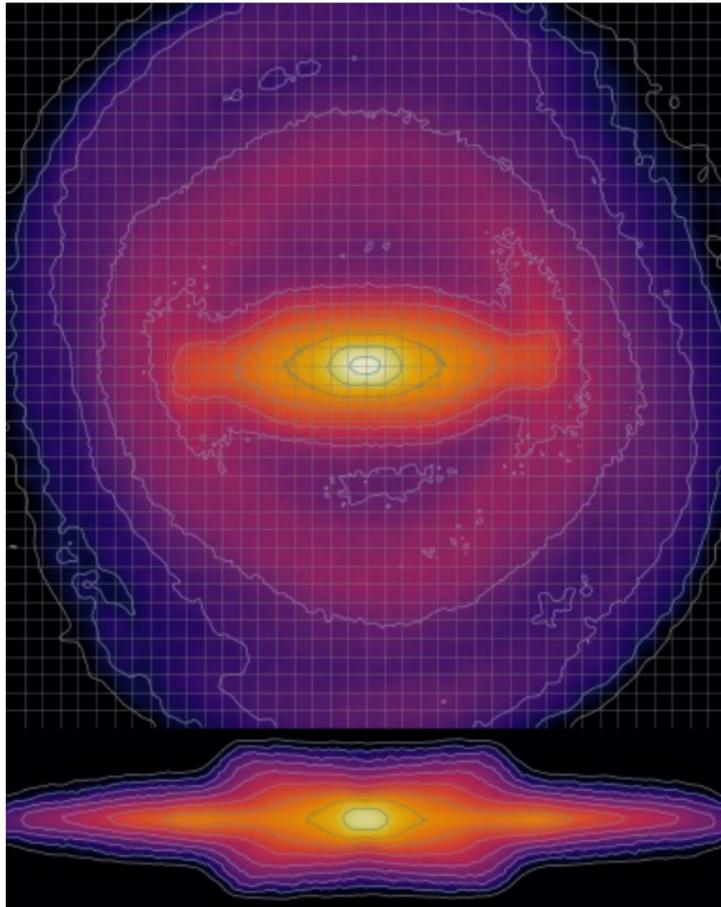
# The turbulent history of a bar-unstable galaxy model

forms a boxy/peanut structure

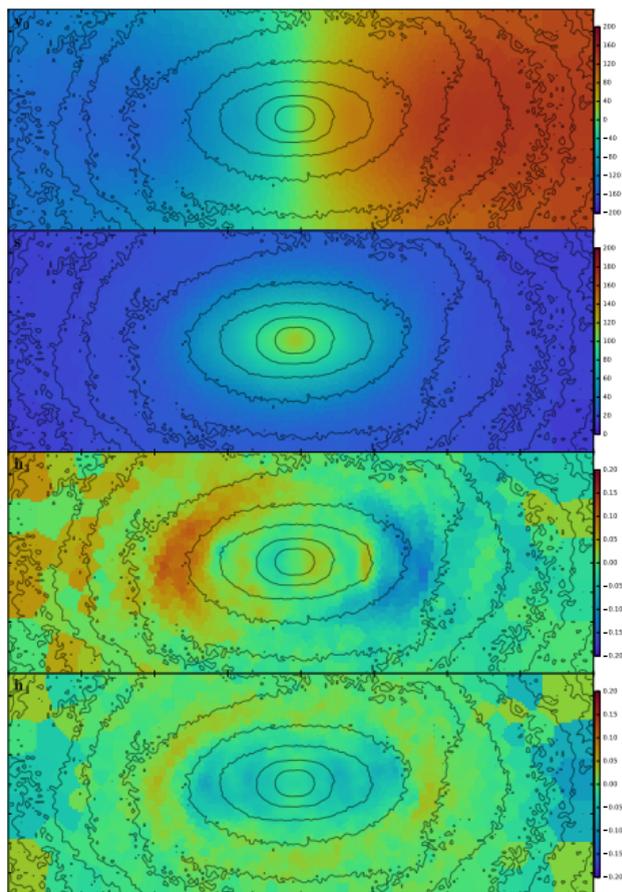


# The turbulent history of a bar-unstable galaxy model

grows longer and forms rings

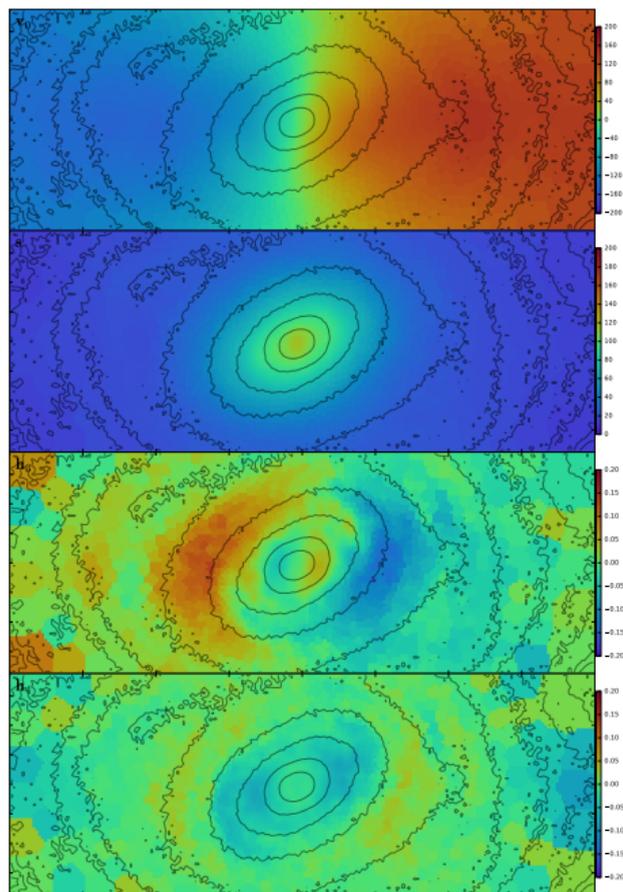


# Kinematic signatures of bars



$i = 45^\circ$

bar parallel to the major axis



bar rotated by  $45^\circ$

## Motion in the rotating frame

$\mathbf{x}$  – position in the rotating frame

$\mathbf{u} \equiv \dot{\mathbf{x}}$  – velocity in the rotating frame

$\mathbf{v} \equiv \mathbf{u} + \boldsymbol{\Omega} \times \mathbf{x}$  – velocity in the inertial frame  
instantaneously aligned with the rotating frame

Equations of motion (symplectic form):

$$\dot{\mathbf{x}} = \mathbf{v} - \boldsymbol{\Omega} \times \mathbf{x}$$

$$\dot{\mathbf{v}} = -\frac{\partial \Phi}{\partial \mathbf{x}} - \boldsymbol{\Omega} \times \mathbf{v}$$

or, in a more familiar form,

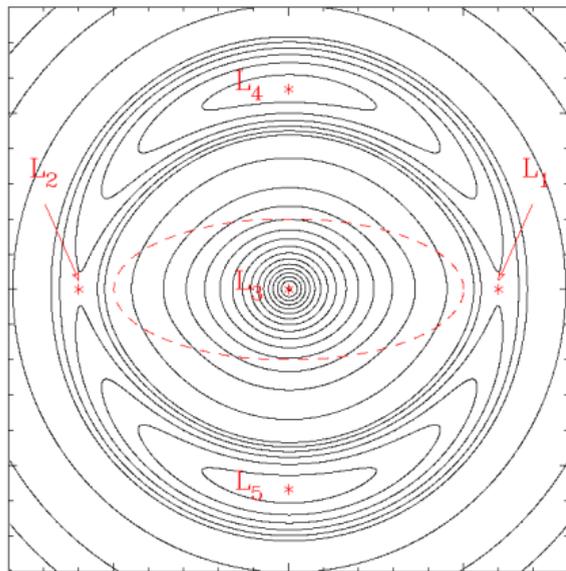
$$\ddot{\mathbf{x}} = -\frac{\partial \Phi}{\partial \mathbf{x}} - \underbrace{2\boldsymbol{\Omega} \times \dot{\mathbf{x}}}_{\text{Coriolis}} - \underbrace{\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{x})}_{\text{\& centrifugal forces}}$$

Effective potential:  $\Phi_{\text{eff}} \equiv \Phi(\mathbf{x}) - \frac{1}{2}|\boldsymbol{\Omega} \times \mathbf{x}|^2$

Jacobi integral:  $E_J \equiv \Phi_{\text{eff}}(\mathbf{x}) + \frac{1}{2}|\dot{\mathbf{x}}|^2$

or  $E_J = E - \boldsymbol{\Omega} \cdot \mathbf{L}$ , where  $\mathbf{L} \equiv \mathbf{x} \times \mathbf{v}$

Effective potential  
 $\Phi - \frac{1}{2}\Omega^2 R^2$  and  
Lagrange points



[from Athanassoula+ 2009]

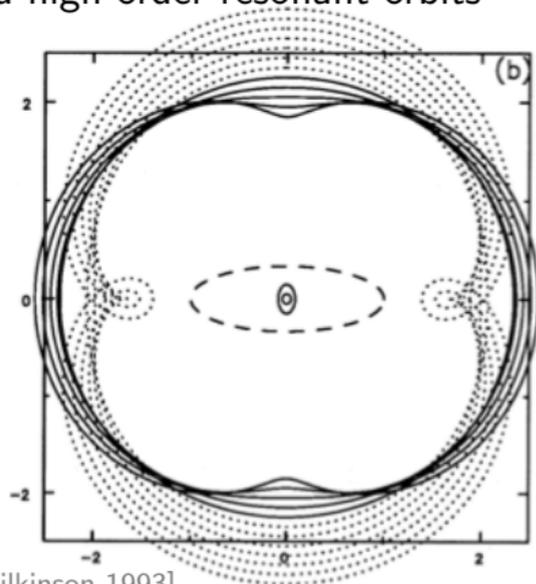
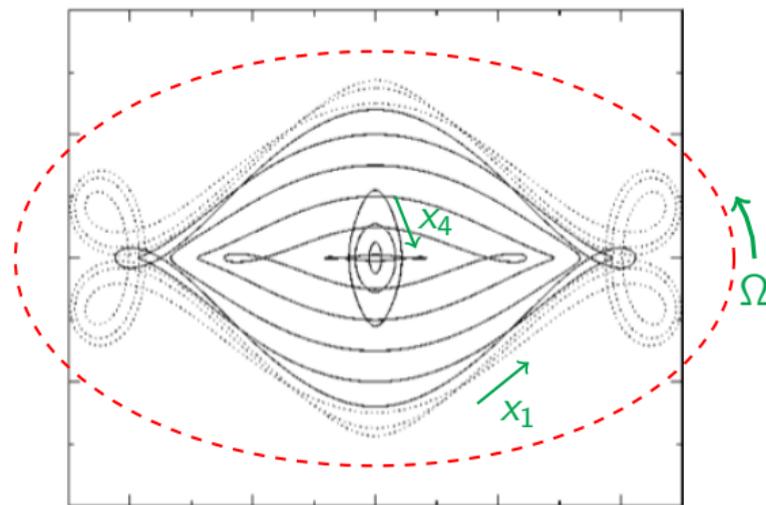
## Orbits in a planar barred potential

Inside corotation radius  $\Omega R_c = v_{\text{circ}}(R_c)$ :

$x_1$ -orbits – prograde, elongated along the bar, support its shape

$x_2$ -orbits (prograde) and  $x_4$  (retrograde), perpendicular to the bar

Outside corotation – no bar-supporting (elongated) orbits;  $x_1$  family now perpendicular to the bar; many chaotic and high-order resonant orbits



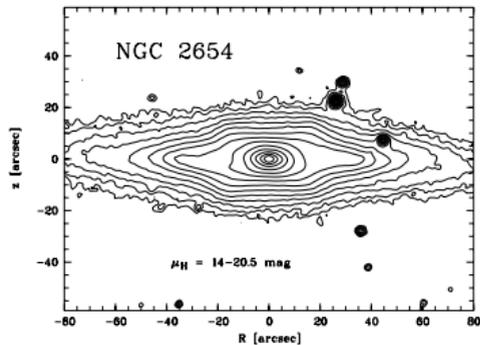
[from the review by Sellwood & Wilkinson 1993]

## 3D structure of bars

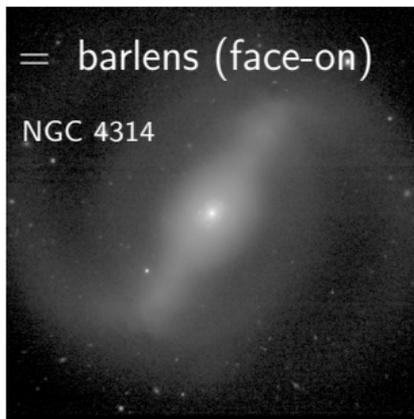
Bars often buckle vertically from the disk plane, but only in the inner part where the planar orbits are unstable;

shorter and vertically thick part is associated with boxy/peanut (B/P) bulges, and the longer and thinner component can be seen in face-on barlens galaxies [Athanasoula 2005, 2013].

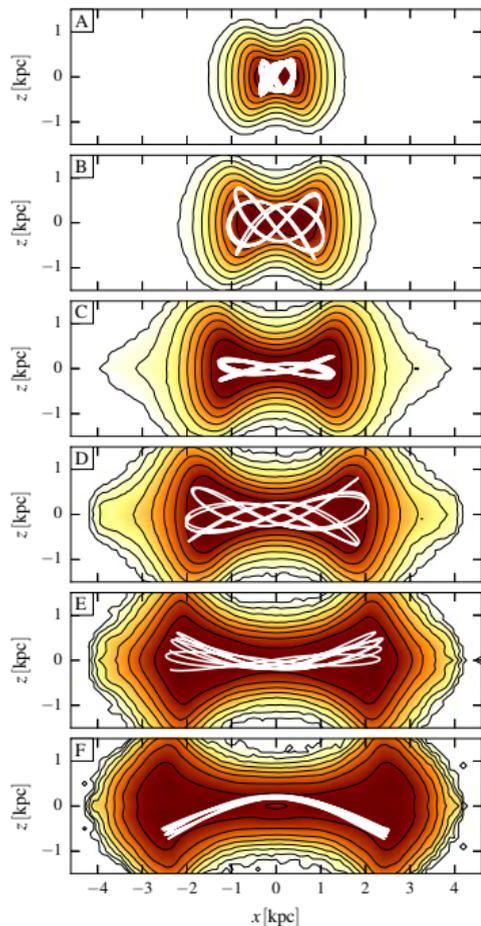
boxy/peanut bar (edge-on) = barlens (face-on)



[Lütticke+ 2000]



[Laurikainen+ 2011]



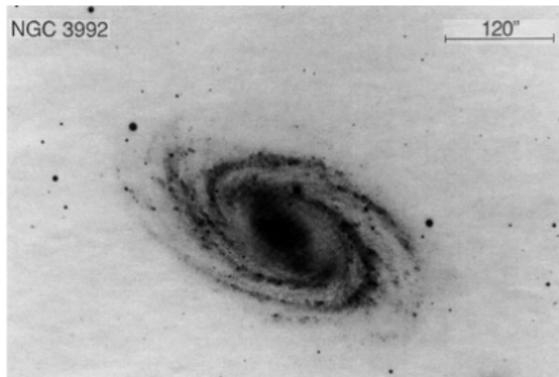
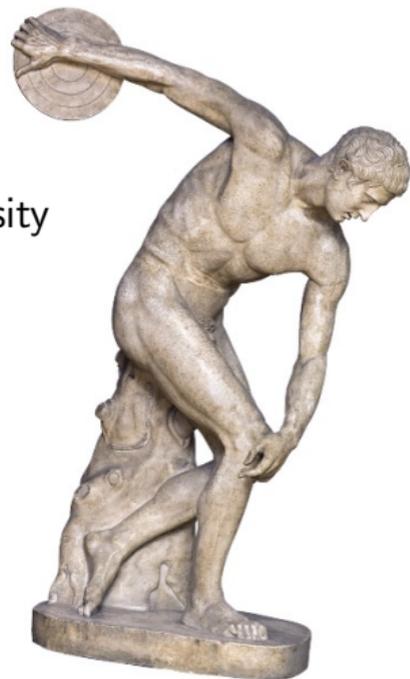
[Portail+ 2015]

# Modelling approaches for barred galaxies: Greek school

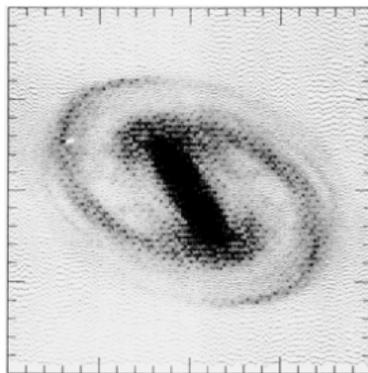
[Contopoulos & Grosbøl 1986, 1988; Patsis+ 1991; Kaufmann & Contopoulos 1996]

2d response models:

- assume parameters for potential, pattern speed, etc.
- construct the network of periodic orbits
- populate nearby orbits and compute their surface density
- compare morphological features with observations
- vary the parameters until a good match is found



observed galaxy



response model

# Modelling approaches for barred galaxies: München school

Made-to-measure method:

introduced by Syer & Tremaine 1996,

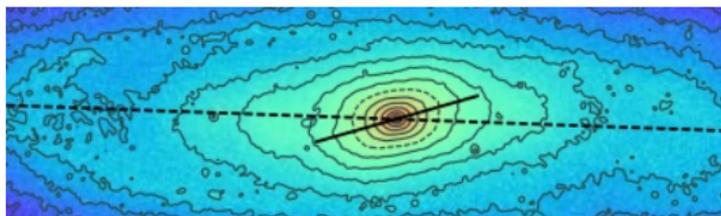
grown up and flourished in Ortwin Gerhard's group

[Bissantz+ 2004, de Lorentzi+ 2007, Portail+ 2015; Blaña+ 2019];

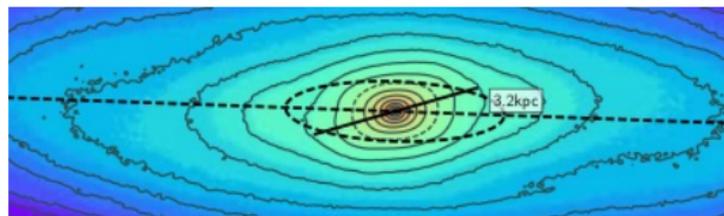
several other implementations exist [Dehnen 2009; Long &

Mao 2012; Hunt & Kawata 2013; Malvido & Sellwood 2015]

Idea: evolve an  $N$ -body model while adjusting particle weights to match the observables (density and kinematics)



observed galaxy (M31)



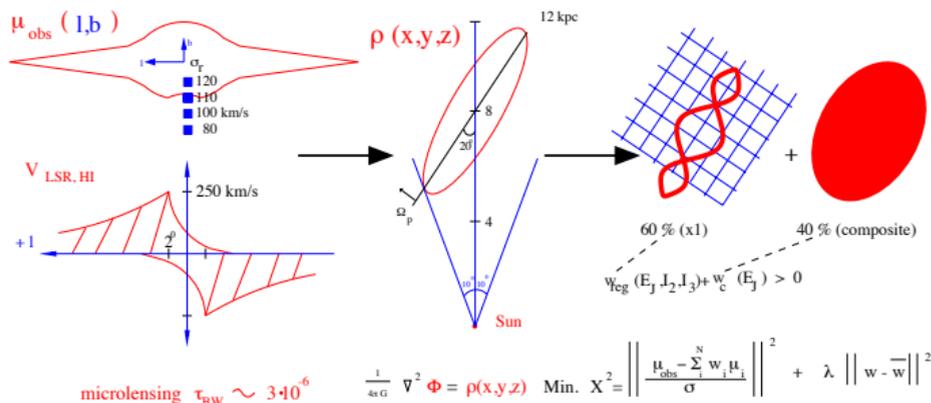
M2M model [Blaña+ 2019]

# Modelling approaches for barred galaxies: Chinese school

Schwarzschild method:

theoretical study of 2d barred galaxies [Pfenniger 1984, 1985],  
application to MW bar [Zhao 1996; Häfner+ 2000; Wang+ 2012]:

density taken from deprojected COBE star counts;  
kinematics fitted to a collection of observations  
(BRAVA survey,  $v_{\text{los}}$  and PM in Baade's window, etc.);  
other constraints: microlensing depth, gas terminal velocities.



[Zhao 1996]



## Schwarzschild method for bars: challenges

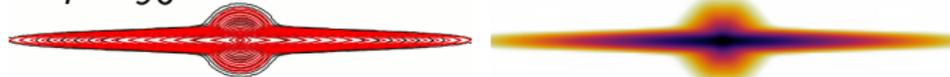
- ▶ Need a 3d density profile and a corresponding potential;
- ▶ Initial conditions need to cover all orbit families;
- ▶ Orbit integration in the rotating frame;
- ▶ Many orbits are chaotic  $\Rightarrow$  bad for self-consistency;
- ▶ Density becomes [nearly] axisymmetric outside corotation;
- ▶ Extra free parameter ( $\Omega_{\text{bar}}$ ) in model search.

# Uncertainties and biases in deprojection

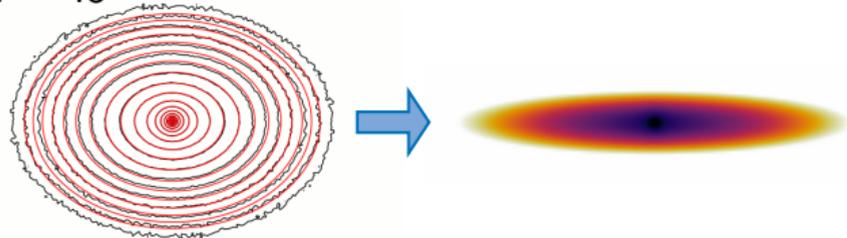
Deprojection is **not unique** even in the axisymmetric (except edge-on) case!

Multi-Gaussian expansion gives only one possible deprojection, but not necessarily a good one.

$i = 90^\circ$



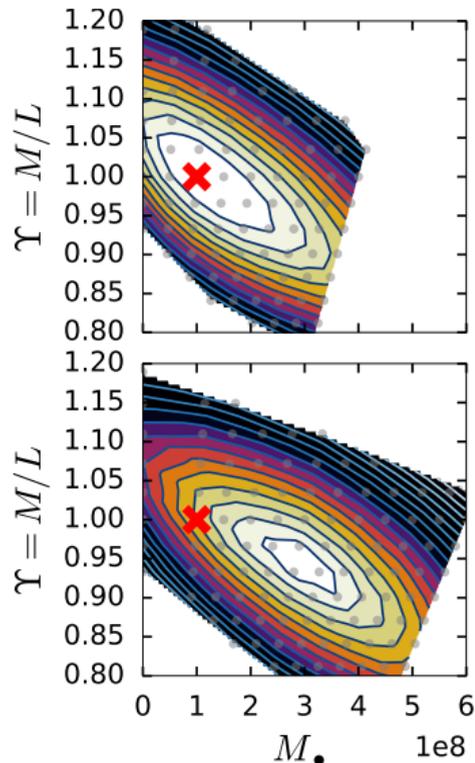
$i = 45^\circ$



actual density profile

deprojected from MGE

[Kochanek & Rybicki 1996;  
Gerhard & Binney 1996]



# Uncertainties and biases in deprojection

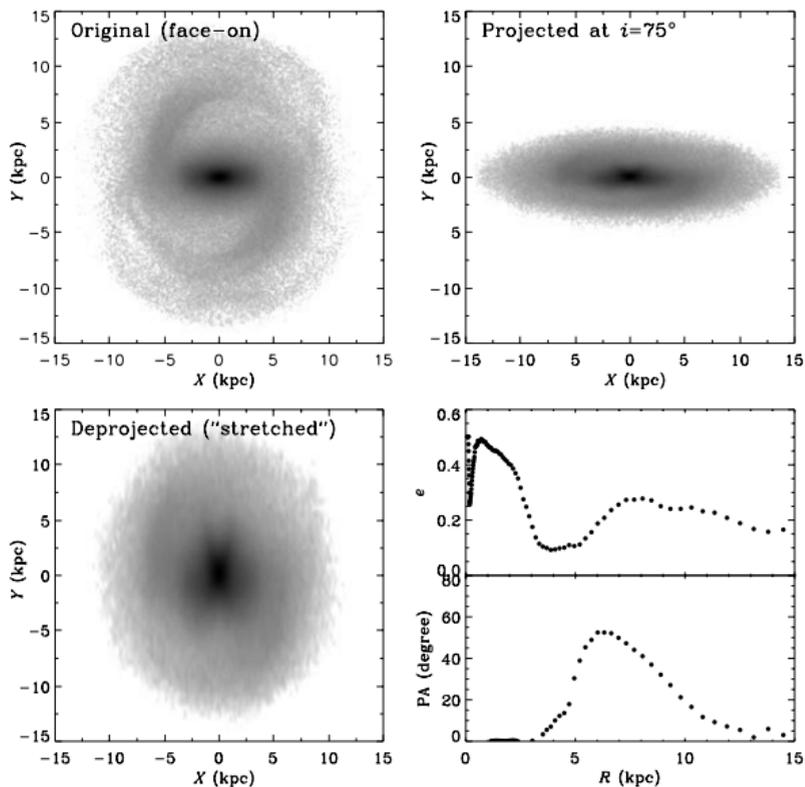
The problem obviously becomes much worse for non-axisymmetric galaxies.

Need an algorithm for systematically exploring the range of possible 3d shapes and orientations consistent with the observed photometry.

Some work has been done earlier but is not widely known...

[Romanowsky & Kochanek 1997; Magorrian 1999; Chakrabarty 2010]

IMFIT [Erwin 2015] can use 3d density profiles and compute projections during image fitting.



[Zou, Shen & Li 2014]

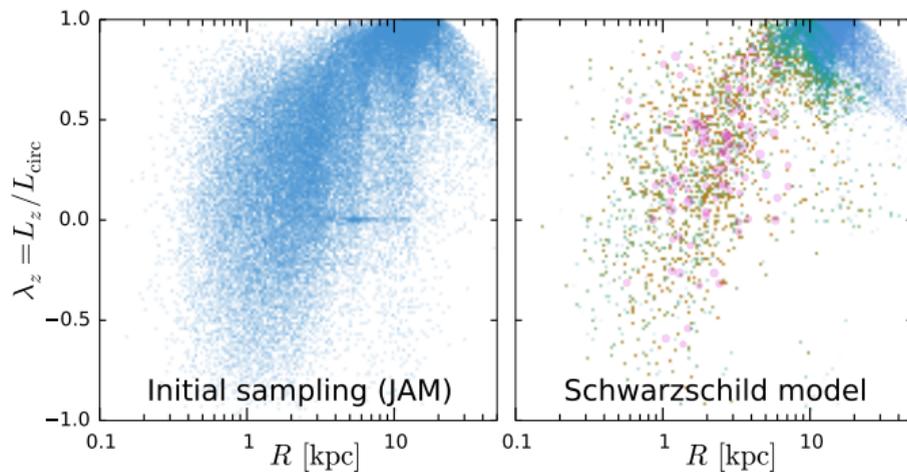
Example of artifacts in bar deprojection

## Initial conditions for orbit integration

**Traditional approach:** use one or more regular “start spaces” (e.g., use several dozen energy levels, and drop orbits from these equipotential surfaces at regular intervals of  $\theta, \phi$ )

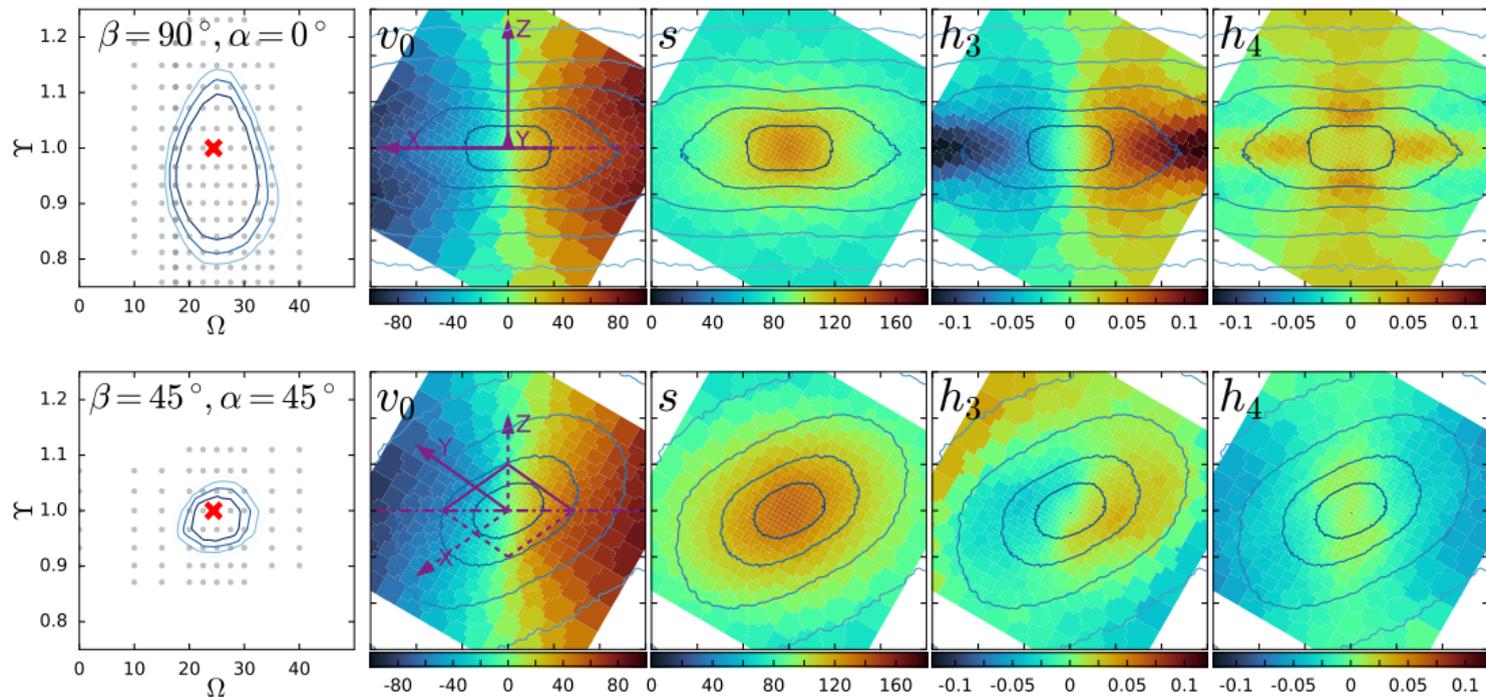
- possible artifacts due to regular sampling and coarse grids
- difficult to ensure that all orbit families are included

**Alternative approach:** sample positions randomly and uniformly from the 3d density profile, and use an auxiliary method (Eddington inversion or its anisotropic generalizations, or Jeans equations) to randomly assign velocity at each point.



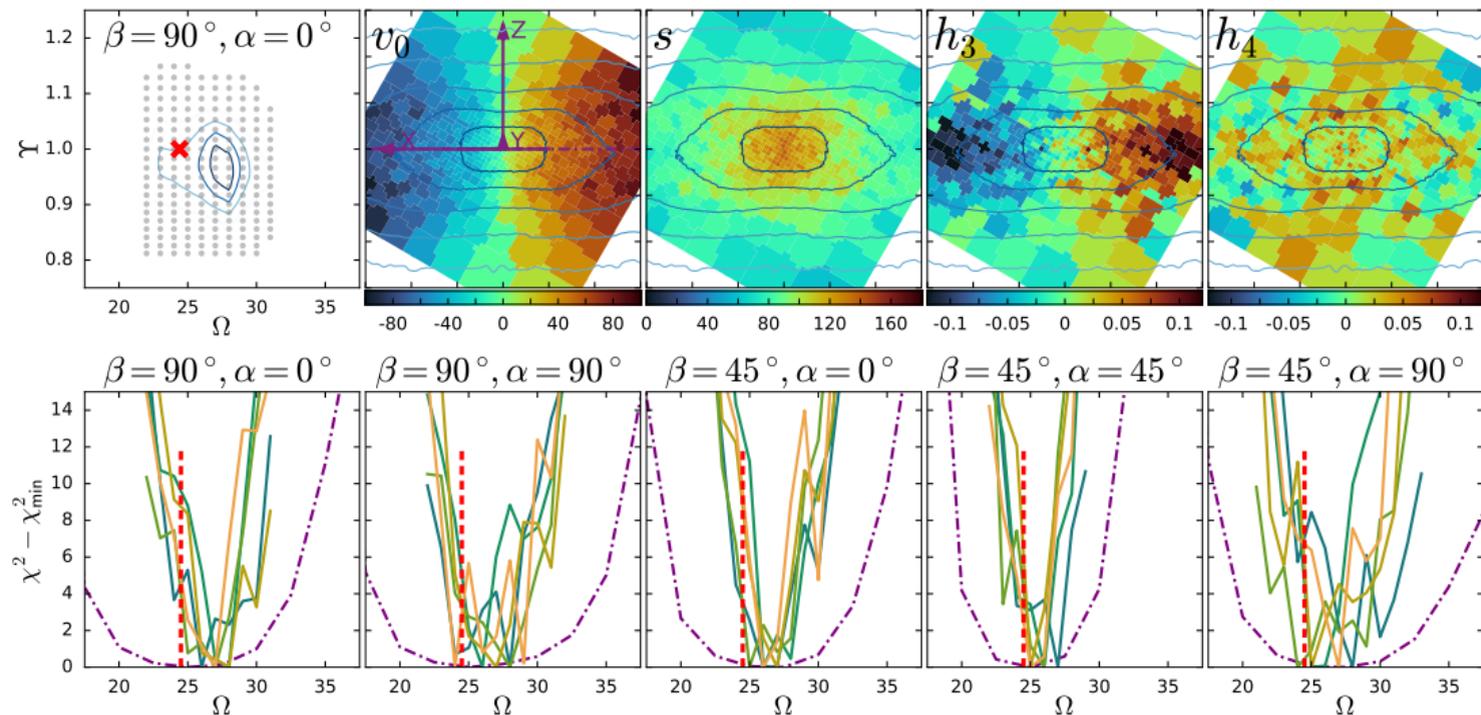
# Recovery of bar pattern speed in Schwarzschild models

Mock data from a barred  $N$ -body model of the Milky Way [Fragkoudi+ 2017]



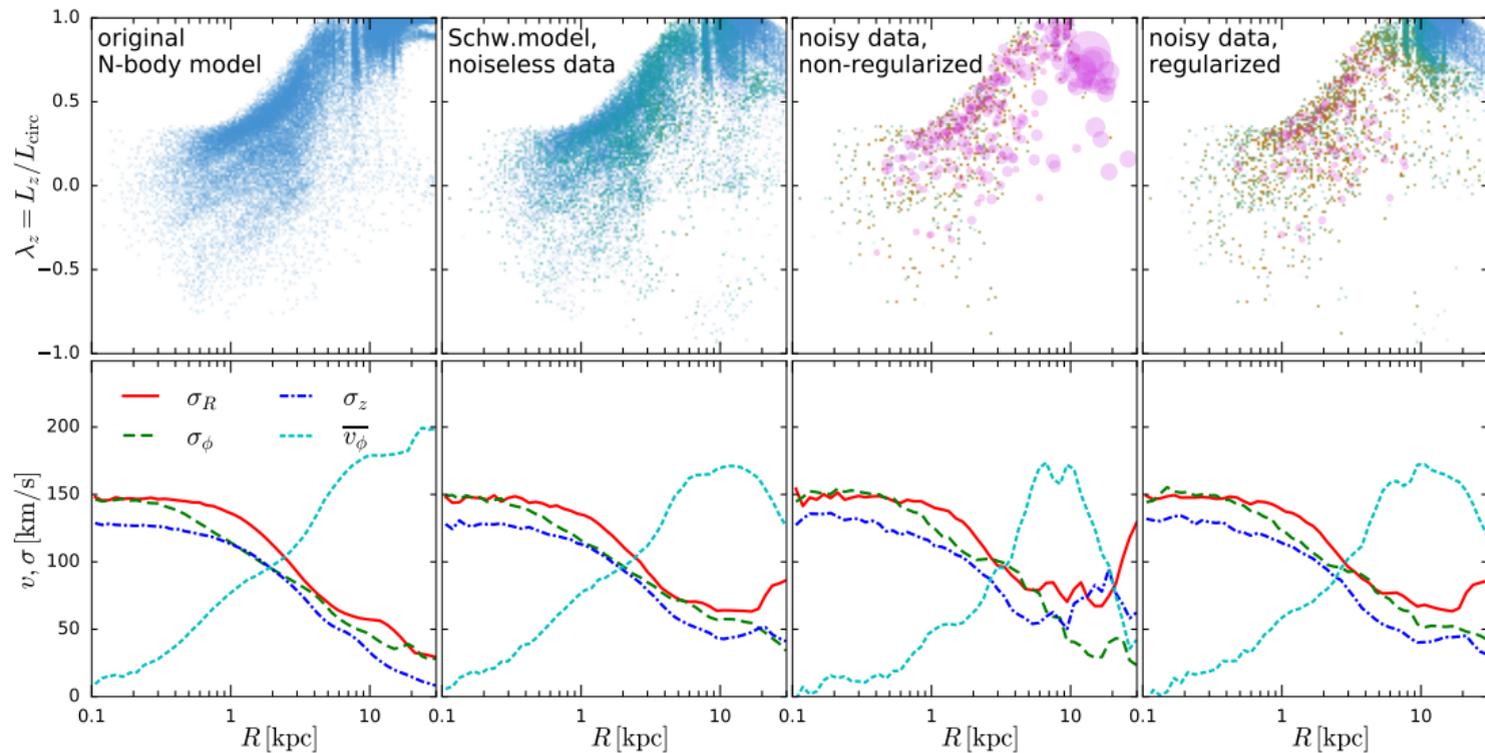
# Recovery of bar pattern speed in Schwarzschild models

Mock data from a barred  $N$ -body model of the Milky Way [Fragkoudi+ 2017]



# Recovery of internal kinematics in Schwarzschild models

Mock data from a barred  $N$ -body model of the Milky Way [Fragkoudi+ 2017]



## Schwarzschild method for bars: the new school

- ▶ Need a 3d density profile and a corresponding potential;  
This remains a difficult and unsolved problem!
- ▶ Initial conditions need to cover all orbit families;  
Use random sampling
- ▶ Orbit integration in the rotating frame;  
EZY!
- ▶ Many orbits are chaotic  $\Rightarrow$  bad for self-consistency;  
Just add more orbits ( $\gtrsim 10 \times N_{\text{constraints}}$ )
- ▶ Density becomes [nearly] axisymmetric outside corotation;  
Only fit  $m \neq 0$  Fourier components inside corotation
- ▶ Extra free parameter ( $\Omega_{\text{bar}}$ ) in model search.  
Optimized nested-grid search, efficient optimization solver

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Kazimir Malevich,  
"Suprematism"