Distribution Function-based dynamical modelling of stellar systems

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Dynamical modelling with discrete tracers

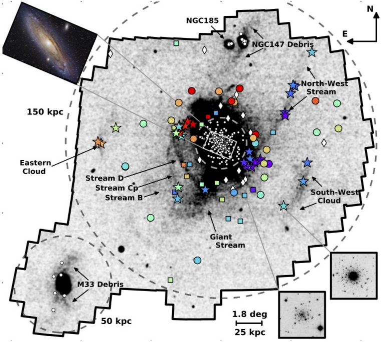
globular clusters
dwarf spheroidals
galactic haloes

$\Bigg\}$ observed positions and velocities of individual stars or other tracers (planetary nebulae, globular clusters) $\Longrightarrow$ gravitational potential: central IMBH, DM halo

ω Cen [credit: NASA]

Sculptor dSph [credit: Legacy Survey]

M31 [PAndAS / Mackey+ 2019]



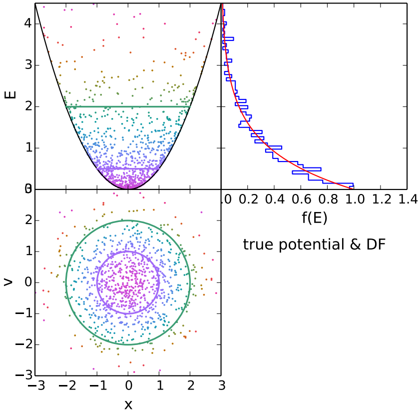
DF modelling

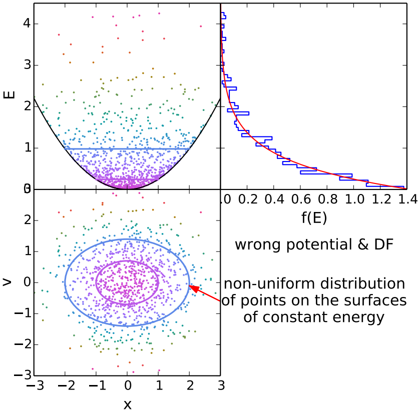
Distribution function of stars (or other tracers) in the 6d phase space: $f(\mathbf{x}, \mathbf{v})$.

By Jeans's theorem, in equilibrium it may only depend on integrals of motion, e.g. actions $\mathbf{J}$, which, in turn, depend on the potential.

By maximising the likelihood of the observed dataset, we determine both f and Φ:

$$\ln \mathcal{L} = \sum_{i=1}^{N_\mathrm{stars}} \ln \mathcal{L}^{(i)} = \sum_{i=1}^{N_\mathrm{stars}} \ln f\left(\mathbf{J}(\mathbf{x}^{(i)}, \mathbf{v}^{(i)}; \Phi)\right).$$



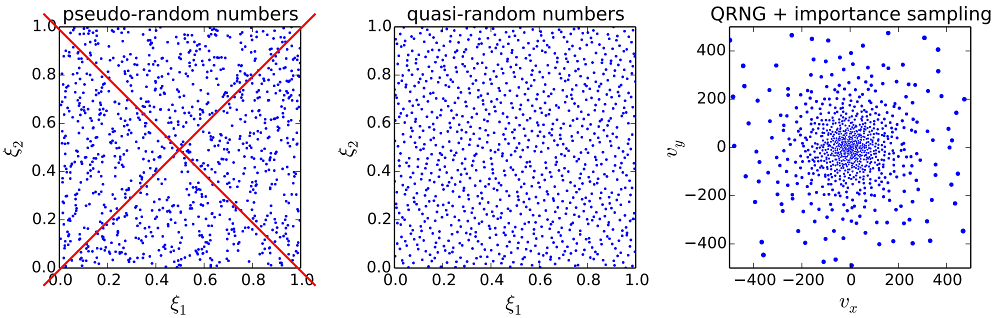


DF modelling

When dealing with incomplete phase-space information (e.g. unknown distance D, proper motions $\boldsymbol{\mu}$, or line-of-sight velocity $v_{\rm los}$), need to marginalise over missing dimensions: $\mathcal{L}^{(i)} = \int_0^\infty \mathrm{d}D\; f\left(\alpha^{(i)}, \delta^{(i)}, D, \mu_\alpha^{(i)}, \mu_\delta^{(i)}, v_{\rm los}^{(i)}\right)$.

Likewise, in case of measurement uncertainties, need to convolve with the error distribution: $\mathcal{L}^{(i)} = \int_{-\infty}^{\infty} \mathrm{d}v_{\rm los}\; \mathcal{N}\left(v_{\rm los} - v_{\rm los}^{(i)}; \epsilon_{v_{\rm los}}^{(i)}\right)\; f\left(\alpha^{(i)}, \delta^{(i)}, D^{(i)}, \mu_\alpha^{(i)}, \mu_\delta^{(i)}, v_{\rm los}\right)$.

In practice, these (multidimensional) integrals are computed using Monte Carlo approach with importance sampling and *quasi*-random (low-discrepancy) numbers.



Previous work on DF modelling

	tracers	α,δ	D	$\mu_{\alpha,\delta}$	v_{los}	potential
Posti & Helmi 2019; Vasiliev 2019; Wang+ 2022	globular clusters in MW halo	+	+	+	+	disc+axi.halo
Correa Magnus & Vasiliev 2022	GC + dSph in MW halo	+	+	+	+	disc+sph.halo
Hattori+ 2021	RR Lyrae in MW halo	+	+	+	−	disc+axi.halo
Pascale+ 2018, 2019, 2024; Arroyo-Polonio+ 2025	stars in dSph	+	−	−	+	spherical
Della Croce+ 2024	stars in GC	+	−	±	±	spherical
Read+ 2021	mock stars in dSph	+	±	±	+	spherical
Gherghinescu+ 2023	mock stars in M31 halo	+	+	+	+	axisym.
this study	mock (dSph or M31)	+	−	−	+	axisym.

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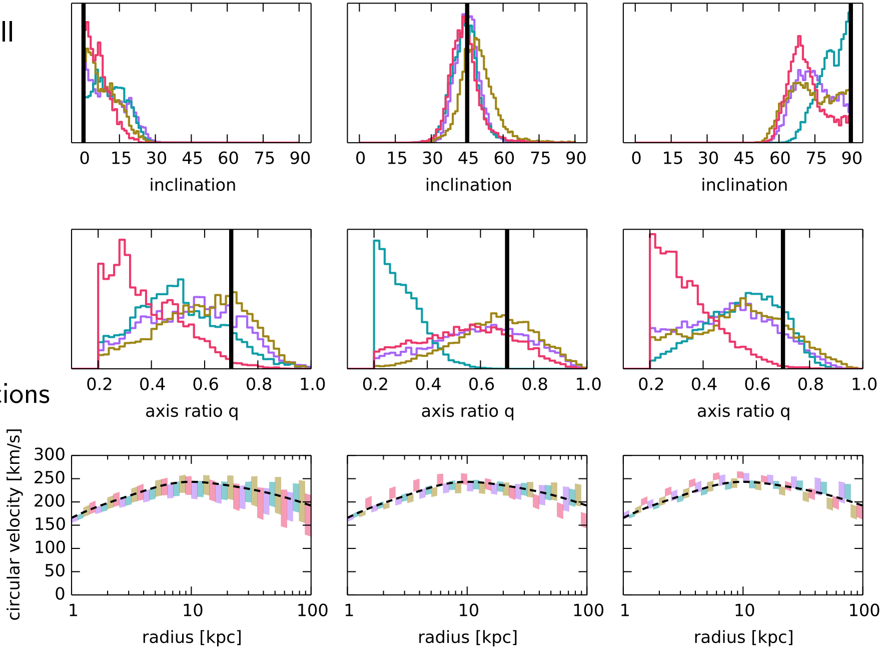
this combination was never explored before

Ingredients

- gravitational potential $\Phi(R, z)$: stellar disc (fixed) + flattened NFW halo $\rho_\mathrm{h}(R, z) = \rho_0\, \chi^{-1}(1+\chi)^{-2}, \quad \chi \equiv \sqrt{R^2 + (z/q)^2}/r_\mathrm{scale}$ with axis ratio q.
- double-power-law DF [Posti+ 2015]:
$$f(\mathbf{J}) = \frac{M_0}{(2\pi J_0)^3}\left[1 + \frac{J_0}{h(\mathbf{J})}\right]^{\alpha}\left[1 + \frac{g(\mathbf{J})}{J_0}\right]^{(\alpha-\beta)}\left(1 + \chi \tanh\frac{J_\phi}{J_{\phi,0}}\right), \text{ where}$$
$$g(\mathbf{J}) = g_r J_r + g_z J_z + (3 - g_r - g_z)\,|J_\phi|\,,$$
$$h(\mathbf{J}) = h_r J_r + h_z J_z + (3 - h_r - h_z)\,|J_\phi|\,.$$
- several realizations of $\mathcal{O}(10^3)$ tracers with 3d (or more) phase-space coordinates $(\alpha, \delta, v_\mathrm{los}, \dots)$, drawn from an equilibrium model (DF+Φ) or from metal-poor accreted stars in Auriga galaxy #23.
- three choices of inclination angle (0°, 45°, 90°).
- explore the parameter space (3 for Φ + 9 for DF + inclination) using EMCEE [Foreman-Mackey+ 2013].

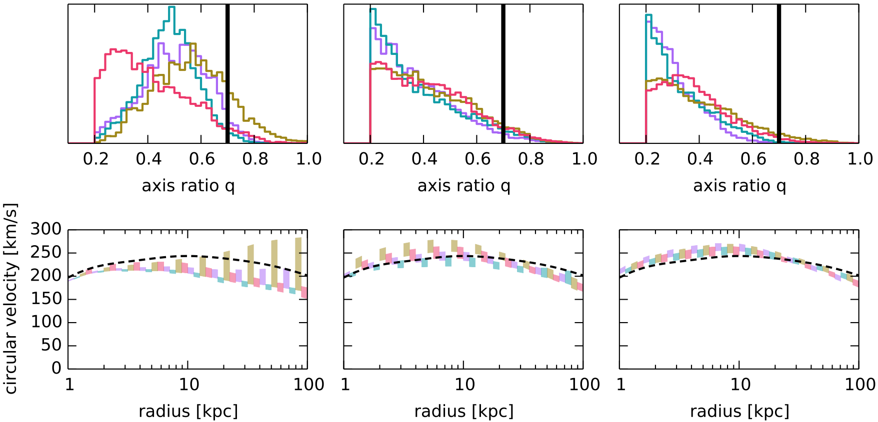
Results (idealised mocks)

- inclination is well constrained and pretty accurate
- axis ratio is not well constrained and biased low
- mass profile is well recovered
- large variation between realizations



Results (Auriga mocks)

- run at fixed inclination
- axis ratio is poorly constrained and even more biased towards low q
- mass profile is biased low for face-on, high for edge-on orientations



Discussion

Q: why is it not possible to constrain the potential flattening q?

A: not enough information: observational constraints are 3d $f(x, y, v_z)$, want to infer *both* the DF in the 3d action space $f(\mathbf{J})$ *and* $\Phi(R, z)$... good luck!

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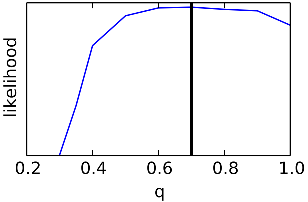
A: unlikely; repeated the analysis with the much more flexible Schwarzschild orbit-superposition method, got a similar result – wide range of allowed q values.



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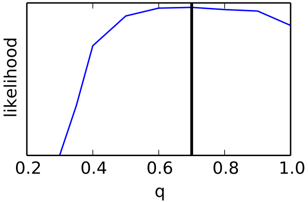

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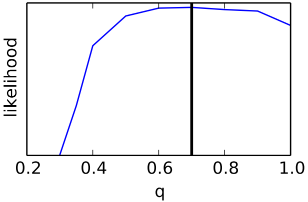

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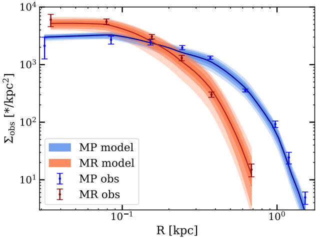
A: most likely because of accumulation of slight inaccuracies in the Stäckel approximation for actions (which increase for more non-spherical models).

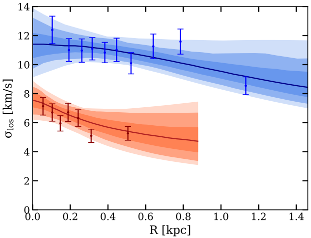
Q: so, what to do?

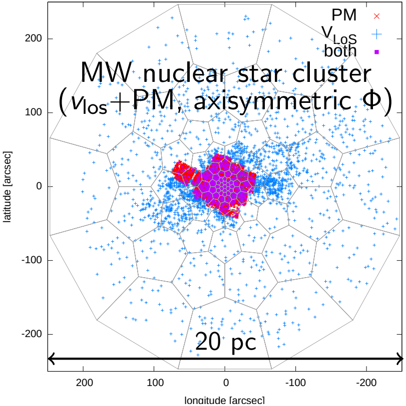
A: use spherical models and forget about trying to constrain the flattening. (this advice applies *only* to the case of 3d obs.data and when the potential is not dominated by observed stars).

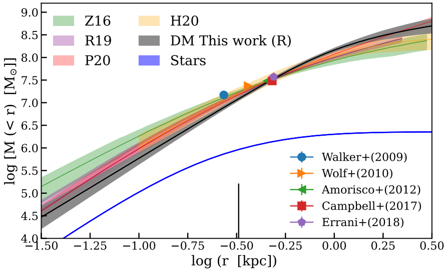
Other examples of DF fitting

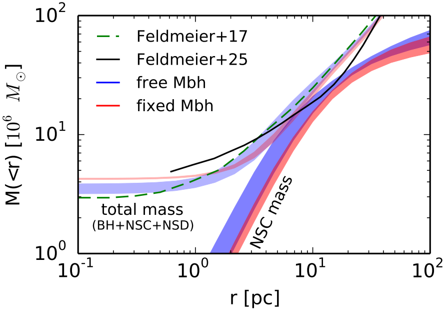
Two-population DF-based model of Scupltor dSph
(v_{los} only, spherical potential)











[Arroyo-Polonio+ 25]